Joint Reconstruction with Parallel Level Sets

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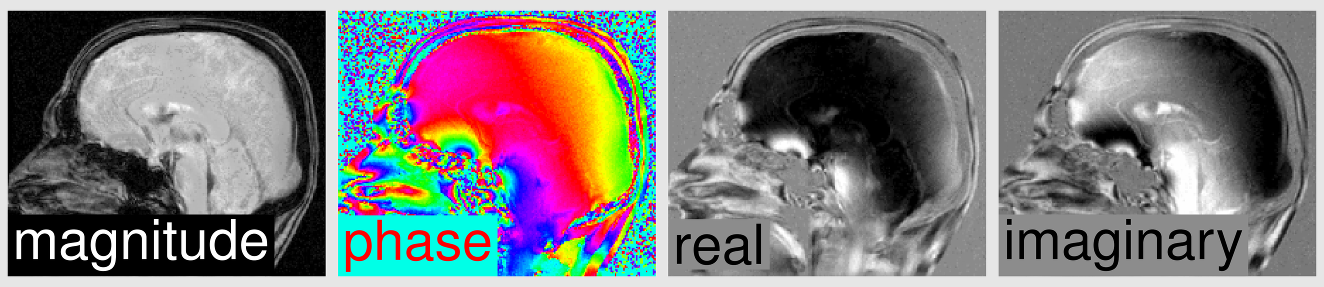
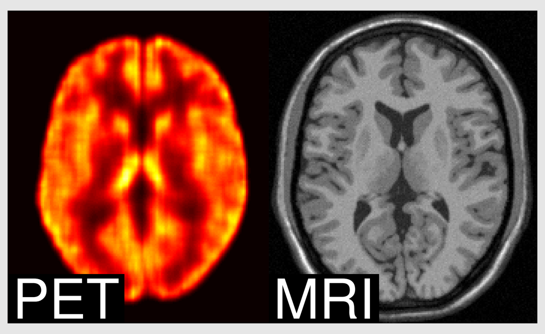
Parallel Level Sets in MRI



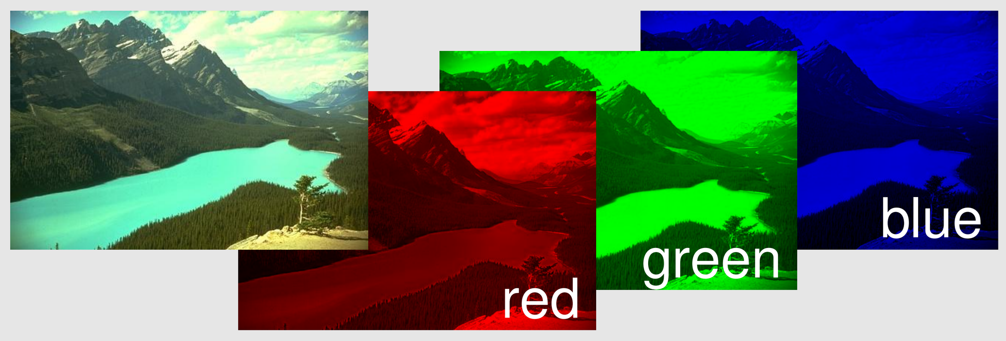
Magnetic resonance imaging (MRI) images are complex [1]. Their real and imaginary parts show similar structures.

... PET-MRI

Positron emission tomography (PET) reconstruction can be improved by anatomical information of MRI which can be acquired by simultaneous PET-MRI scanners [2].



... RGB Images



The main structures of the image are visible in all colour channels [3].

Method

Aim: Design a penalty functional $\mathcal{R}(u,v)$ for *joint reconstruction* [4] of the form

$$\operatorname*{argmin}_{u,v} \|A(u,v) - g\|^2 + \lambda \mathcal{R}(u,v)$$

which enforces similar structures in u and v.

Proposed Solution: Two vectors x and y are *parallel* iff

$$d(x,y) := \varphi\Big(\psi(\|x\|\,\|y\|) - \psi(|\langle x,y\rangle|)\Big)$$

is zero where φ, ψ are increasing functions.
As similar shapes correspond to parallel level sets we can enforce similarity by minimizing

$$\mathcal{R}(u,v) := \int d(\nabla u, \nabla v) \mathrm{d}x. \qquad (1)$$

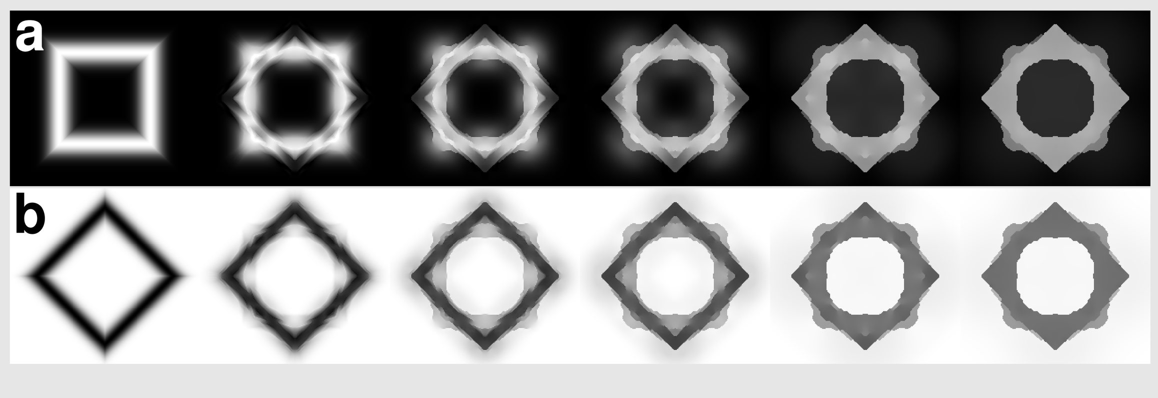
A normalized version of (1) was used for joint reconstruction in geosciences [5] and for multi-modality image registration [6].
The Gâteaux derivative of $\mathcal{R}$ with respect to u and v with zero Neumann boundary conditions takes the form

$$D\mathcal{R}_{u,v} = -\operatorname{div}\left[\begin{pmatrix}\kappa(u,v) & \tau(u,v)\\ \tau(u,v) & \kappa(v,u)\end{pmatrix} \nabla \begin{pmatrix} u \\ v\end{pmatrix}\right]$$

where the diffusivities κ and τ couple u and v.
The extension to multiple data sets can be done by pairwise comparison.
A heuristic change modifies (1) so that it reduces to smooth total variation when one of the images is flat,

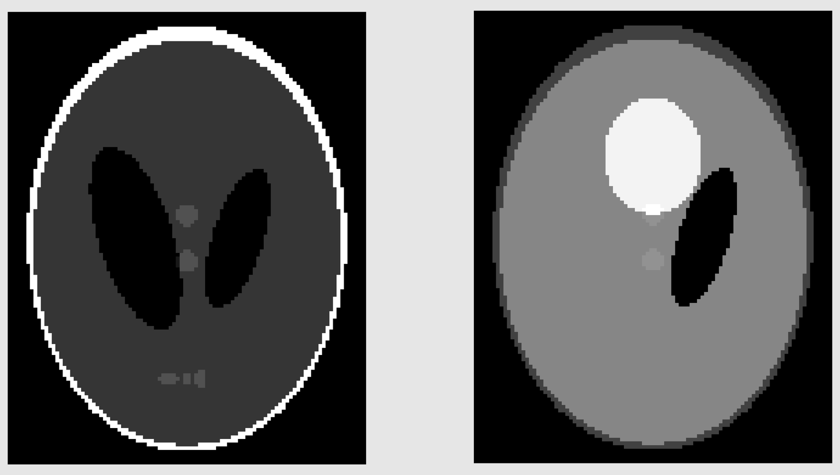
$$\mathcal{R}(u,v) = \int [\![\nabla u]\!]\, [\![\nabla v]\!] - [\![\langle \nabla u, \nabla v\rangle]\!]\, \mathrm{d}x \qquad (2)$$

with $[\![x]\!] := \sqrt{x^2+1}$ and u and v scaled to be dimensionless [3].

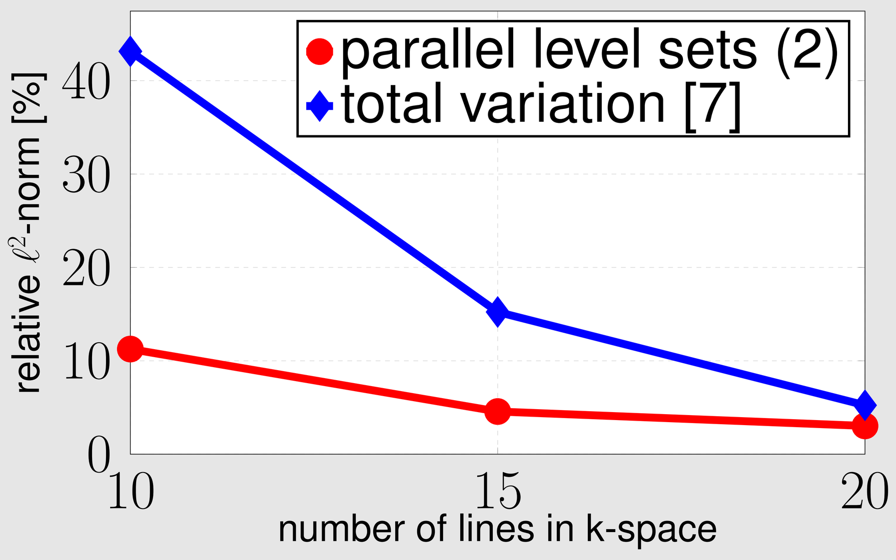


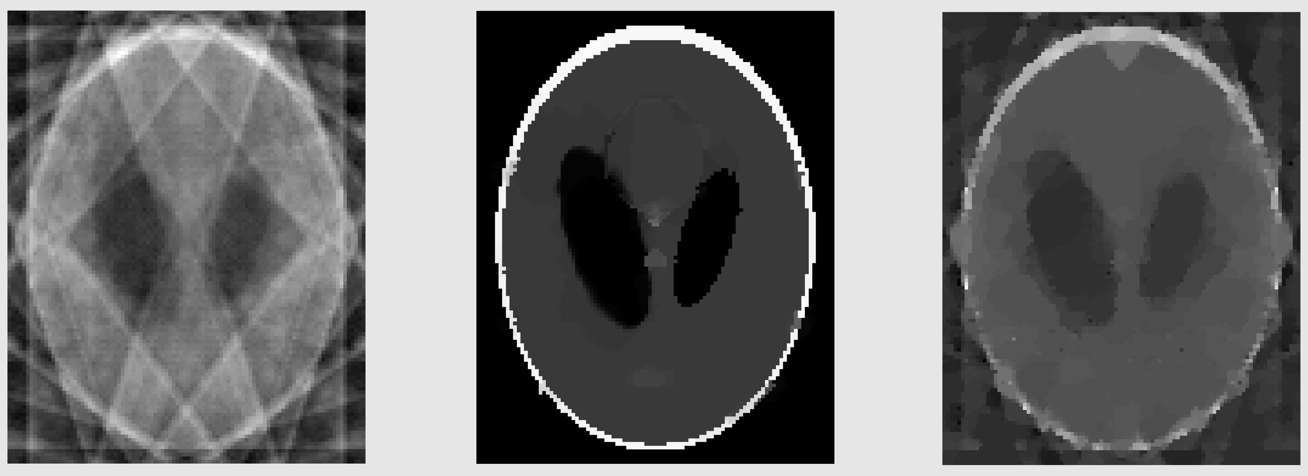
The figure shows two test images that evolve by minimizing equation (2) towards parallel level sets (a), (b) [3].

Undersampled MRI with Side Information



phantom side information

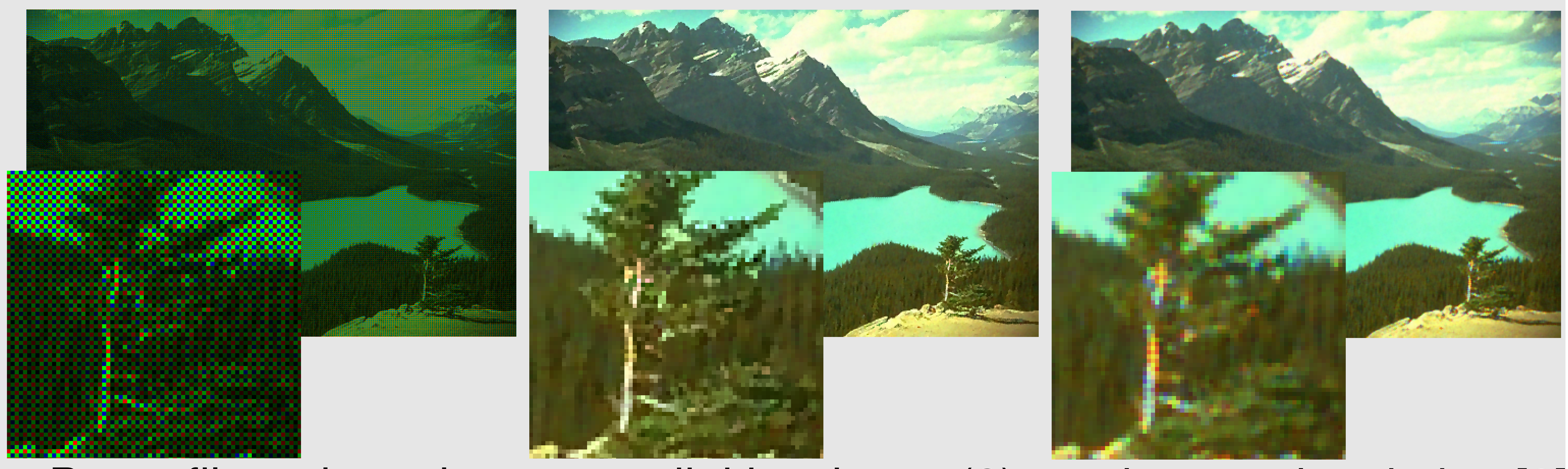




minimal energy parallel level sets (2) total variation [7]

Exploiting the similarity between the phantom and the side information yields better reconstructions even in those areas where the side information is missing. The introduced artefacts are minor compared to the gain in image quality.

Demosaicking of RGB Images



Bayer filtered + noise parallel level sets (2) colour total variation [8]

While colour total variation shows colour artefacts due to the missing coupling of information in the colour channels the parallel level sets based method reconstructs sharp images with consistent colours [3].

Conclusions

The proposed method is able to combine information about shapes of several images. Exploiting these shared features all images can be enhanced. A next step is the application to simultaneous PET-MRI reconstruction.

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References

[1] D. W. McRobbie, E. A. Moore, M. J. Graves, and M. R. Prince, *MRI - From Picture to Proton.* Cambridge University Press, 2006.

[2] D. W. Townsend, "Multimodality Imaging of Structure and Function," *Physics in Medicine and Biology*, vol. 53, no. 4, pp. R1–R39, 2008.

[3] M. J. Ehrhardt and S. Arridge, "Vector-Valued Image Processing by Parallel Level Sets," *submitted to: IEEE Transactions on Image Processing*, 2013.

[4] E. Haber and D. W. Oldenburg, "Joint Inversion: A Structural Approach," *Inverse Problems*, vol. 13, pp. 63–77, 1997.

[5] L. A. Gallardo and M. A. Meju, "Characterization of Heterogeneous Near-Surface Materials by Joint 2D Inversion of DC Resistivity and Seismic Data," *Geophysical Research Letters*, vol. 30, no. 13, p. 1658, 2003.

[6] E. Haber and J. Modersitzki, "Intensity Gradient based Registration and Fusion of Multi-Modal Images," in *Medical Image Computing and Computer-Assisted Intervention - MICCAI*, vol. 46, no. 3. Springer-Verlag, 2006, pp. 726–733.

[7] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging," *Magnetic Resonance in Medicine*, vol. 58, no. 6, pp. 1182–1195, 2007.

[8] P. Blomgren and T. F. Chan, "Color TV: Total Variation Methods for Restoration of Vector-Valued Images." *IEEE Transactions on Image Processing*, vol. 7, no. 3, pp. 304–309, 1998.

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