Generative Regularizers for Inverse Imaging Problems

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Motivation

- Many inverse problems (e.g. MRI) can be solved via **variational regularization**

$$x^* \in \arg\min_x \{D(Ax, y) + R(x)\}$$

- How to get a **good regularizer** R**?**

Generative Regularizers

- Given a generative model $G : Z \to X$ (e.g. VAE, GAN), one can define a **generative regularizer**

$$R(x) = \inf_z \left\{ \frac{1}{2}\|x - G(z)\|_2^2 + S(z) \right\}$$

- A variant with **hard constraints** has been used in [1]

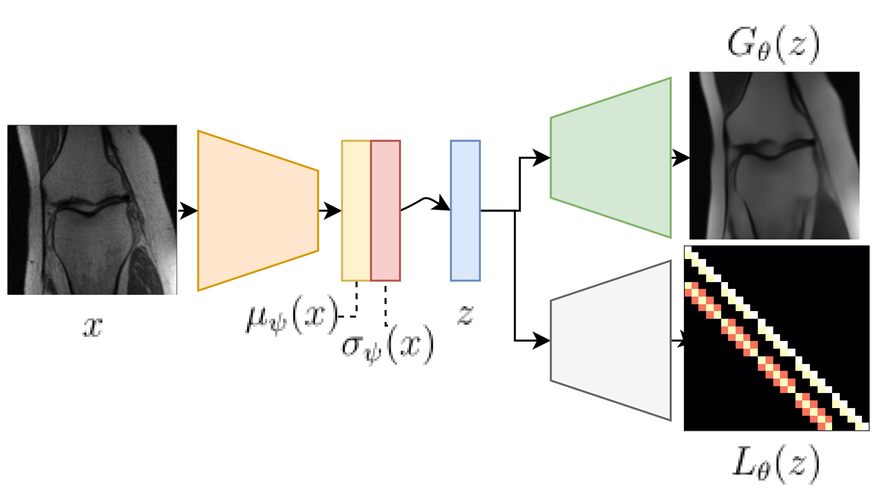
$$R(x) = \inf_z \iota_{\{0\}}(x - G(z))$$

- In both cases, **only the mean** of the distribution is modelled.

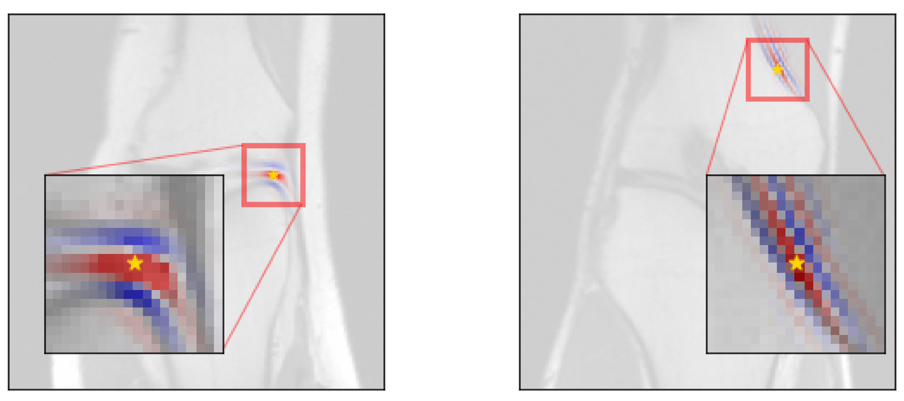
Modelling the Covariance

- Motivated by [2] we use the regularizer

$$R(x) = \inf_z \left\{ \log\det(\Sigma(z)) + \frac{1}{2}\|x - G(z)\|^2_{\Sigma^{-1}(z)} + \frac{1}{2}\|z\|_2^2 \right\}$$

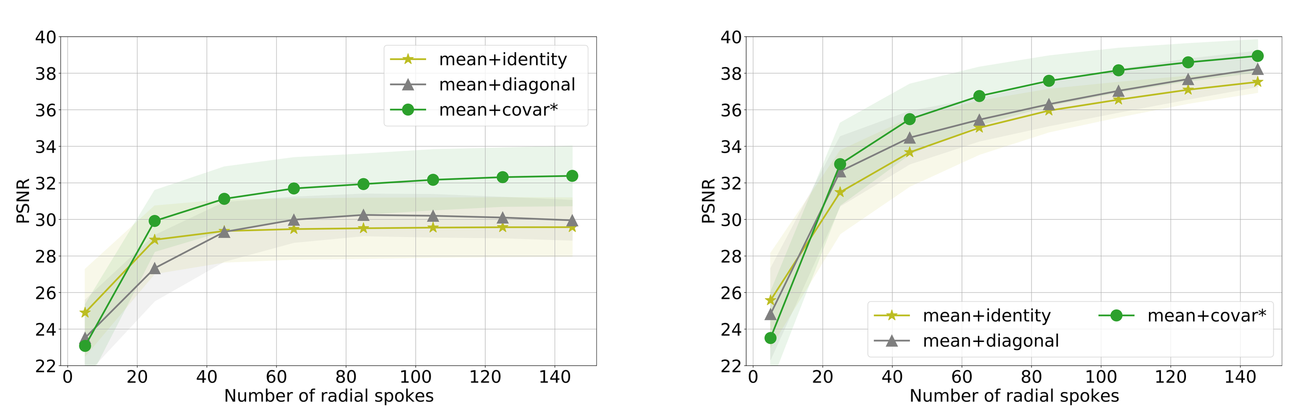


- Visualization of learned positive and negative covariance.



Comparison: Covariance Models

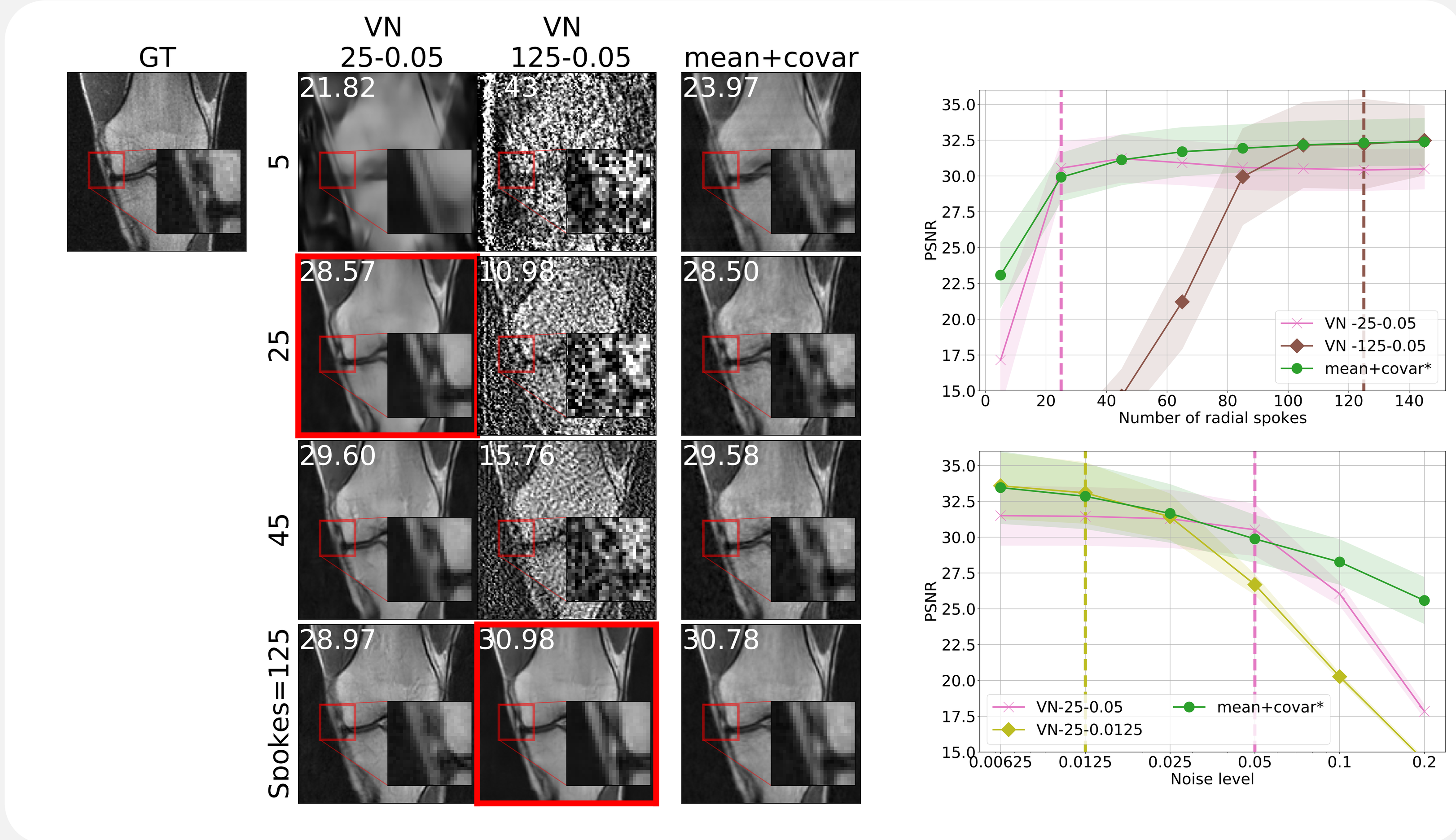
- Compare: constant diagonal (identity), varying diagonal (diagonal) and proposed (covar)



- In any case the proposed model appears superior.

Comparison: End-to-end Learning

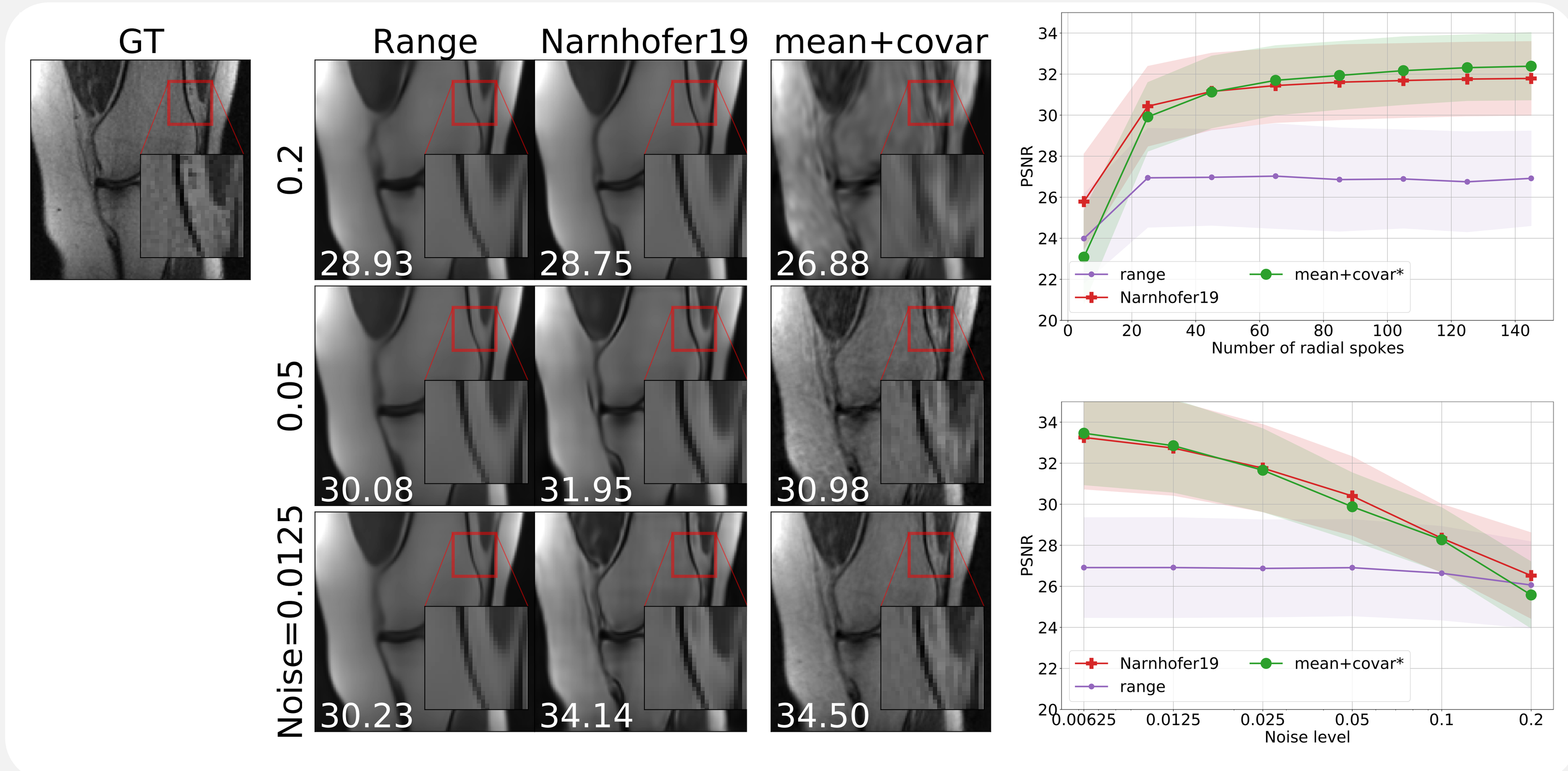
- Compare to Variational Network (VN) [3] trained for specific sampling and noise (indicated with red frame and dashed lines).



- Similar peak performance but proposed model **generalizes better** to unseen settings.

Comparison: Other unsupervised methods

- Compare to [1] (Range) which restricts to the range.
- Compare to [4] (Narnhofer19) which uses an Inverse GAN.



- Better than [1]. Similar to [4].
- Both [1] and [4] produce **smoother solutions**.

Conclusions

- Advanced modelling of prior: **covariance**
- Unsupervised model: **no paired data** required
- Learning independent of inverse problem: **generalization**

References

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