$1 + 1 > 2$? Getting More Out of Multi-Modality Imaging

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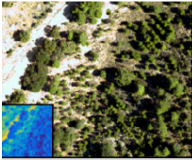
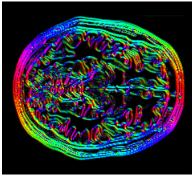
The Faraday Institution

Outline

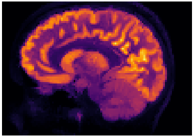
1) Motivation: Examples of Multi-Modality Imaging (**Why?**)



2) Mathematical Models for Multi-Modality Imaging (**How?**)



3) Application Examples: Remote Sensing and Medical Imaging ($1 + 1 > 2$**?**)

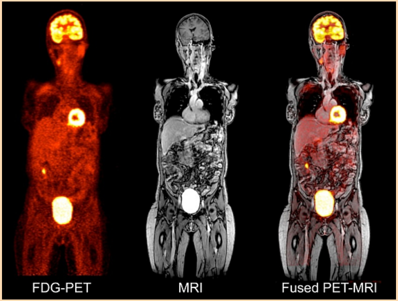


Motivation: Examples of Multi-Modality Imaging

Multi-Modality Imaging Examples

PET-MR

PET-MR (and PET-CT, SPECT-MR, SPECT-CT)



Combine **anatomical (MRI)** and **functional (PET)** information

7 **clinical scanners** in UK

Currently images are just **overlayed**

Challenge: Reduce scanning time, increase image quality, lower dose

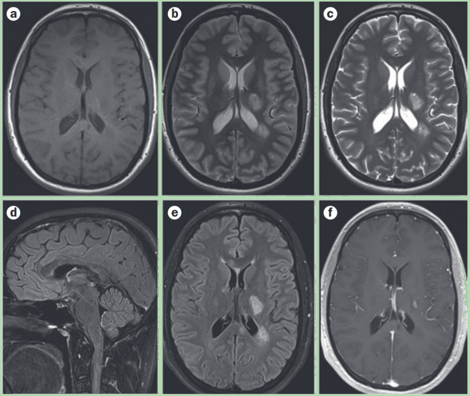
image: Sheth and Gee, 2012

Multi-Modality Imaging Examples

PET-MR

Multi MRI

Multi-Sequence MRI



Rovira et al., Nature Reviews Neurology, 2015

pre-contrast
T_1-weighted (a),
dual-echo T_2 (b, c)
post-contrast
2D T_2 FLAIR (d, e),
T_1-weighted (f)

Standardized MRI protocol
for multiple sclerosis

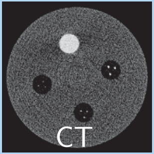
6 scans, total 30 min

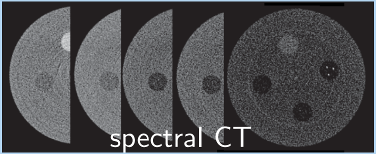
Challenge: Reduce scanning time

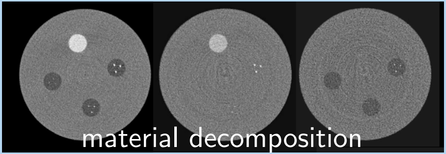
Multi-Modality Imaging Examples

PET-MR | Multi MRI | Spectral CT

Spectral CT







images:
Shikhaliev and Fritz, 2011

Acquisition: energy resolved measurements

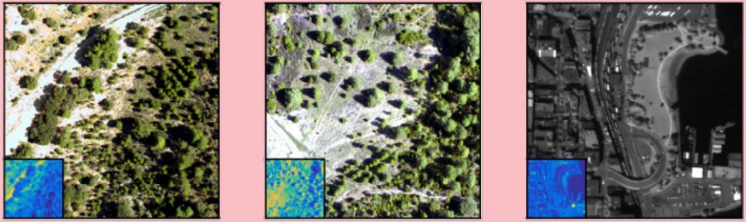
Combination: material information

Challenge: Low dose / high noise in some channels

Multi-Modality Imaging Examples

PET-MR | Multi MRI | Spectral CT | Hyper + optical

Image fusion in remote sensing



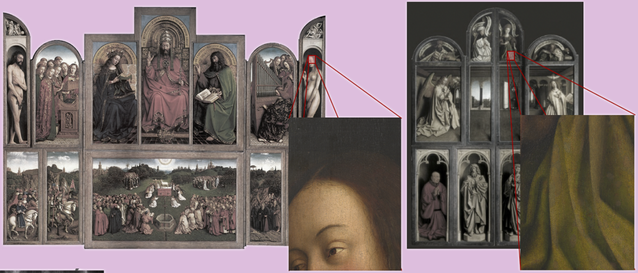
Acquisition: low resolution hyperspectral data (127 channels, $1m \times 1m$) and high resolution photograph ($0.25m \times 0.25m$) acquired **on plane or satellite**, e.g. by NERC Airborne Research & Survey Facility

Challenge: get best of both worlds—high spatial and spectral resolution

Multi-Modality Imaging Examples

PET-MR	Multi MRI	Spectral CT	Hyper + optical	X-ray + optical

X-ray separation for art restauration Deligiannis et al. 2017

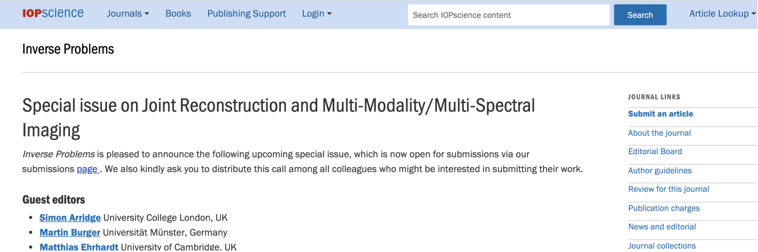


Acquisition: photographs and x-ray images

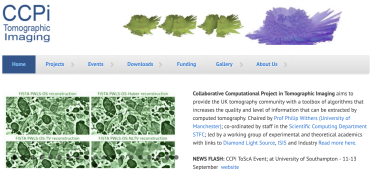
Challenge: separate the x-rays of the doors

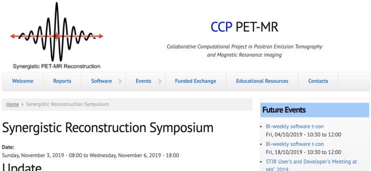
Fairly Large Field

- Regular **sessions at major conferences**: Applied Inverse Problems, SIAM Imaging
- **Symposium** in Manchester in 3-6 Nov 2019
- **Special Issue** in IOP Inverse Problems



- **Collaborative Software Projects**: CCPi (Phil Withers) and CCP PETMR





Mathematical Models for Multi-Modality Imaging

Image Reconstruction

Variational Approach:

$$u^* \in \arg\min_u \Big\{ \mathcal{D}(\mathbf{A}u, b) + \alpha \mathcal{J}(u) + \imath_C(u) \Big\}$$

- $\mathbf{A}$ **forward operator** (often but not always linear), e.g. Radon transform
- $\mathcal{D}$ **data fit**, e.g. least-squares $\mathcal{D}(\mathbf{A}u, b) = \frac{1}{2}\|\mathbf{A}u - b\|^2$, Kullback–Leibler divergence $\mathcal{D}(\mathbf{A}u, b) = \int \mathbf{A}u - b + b\log(b/\mathbf{A}y)$
- $\mathcal{J}$ **regularizer**, e.g. total variation $\mathcal{J}(u) = \mathrm{TV}(u) := \sum_i |\nabla u_i|$ Rudin et al., 1992
- $\imath_C$ **constraints**, e.g. nonnegativity

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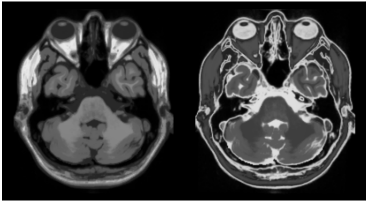
How to include information from other modalities?

Modelling Structural Similarity



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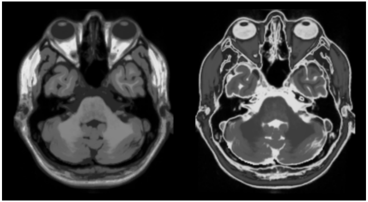

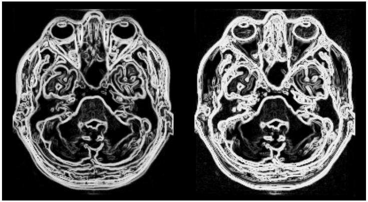

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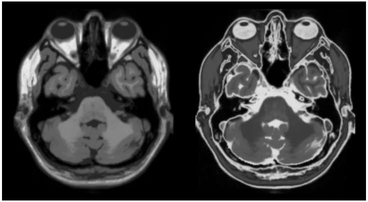
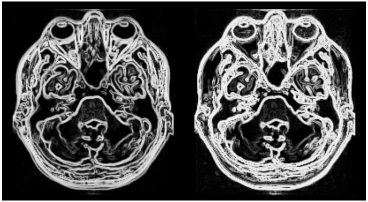




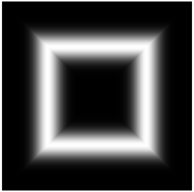
Definition: The **Weighted Total Variation (wTV)** of u is

$$\mathrm{dTV}(u) := \sum_i w_i \|\nabla u_i\|, \quad 0 \leq w_i \leq 1$$

See e.g. Ehrhardt and Betcke '16

- If $c > 0, c < w_i$, then $c\,\mathrm{TV} \leq \mathrm{wTV} \leq \mathrm{TV}$.
- If $w_i = 1$, then $\mathrm{wTV} = \mathrm{TV}$.
- $w_i = \frac{\eta}{\|\nabla v_i\|_\eta}, \quad \|\nabla v_i\|_\eta^2 = \|\nabla v_i\|^2 + \eta^2, \quad \eta > 0$

Modelling Structural Similarity



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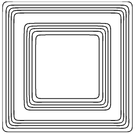
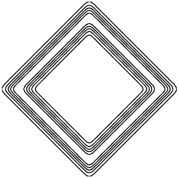

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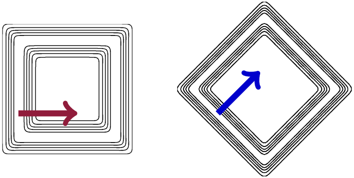

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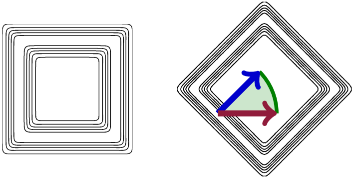
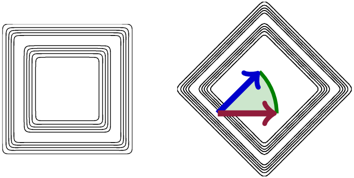


$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Modelling Structural Similarity

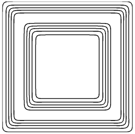


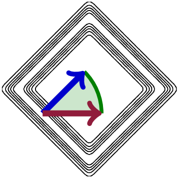
$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u||\nabla v|$$

Definition: Two images u and v are said to have **parallel level sets** or are **structurally similar** (denoted by $u \sim v$) if $\theta = 0$ or $\theta = \pi$, i.e.

$$\nabla u \parallel \nabla v \quad \text{i.e. } \exists\, \alpha \text{ such that } \nabla u = \alpha \nabla v\,.$$

Modelling Structural Similarity





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- Dominant idea in this field
  - Parallel Level Set Prior, e.g. Ehrhardt and Arridge '14
  - Directional Total Variation, e.g. Ehrhardt and Betcke '16
  - Total Nuclear Variation, e.g. Knoll et al. '16
  - Coupled Bregman iterations, e.g. Rasch et al. '18
- Others are: joint sparsity (e.g. wTV), joint entropy, ...

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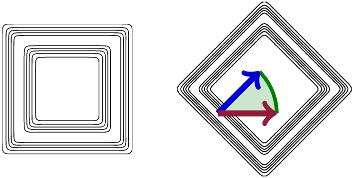

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Directional Total Variation

- Note that if $\|\nabla v\| = 1$, then

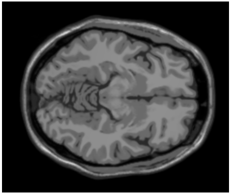
$$u \sim v \quad \Leftrightarrow \quad \nabla u - \langle \nabla u, \nabla v \rangle \nabla v = 0$$

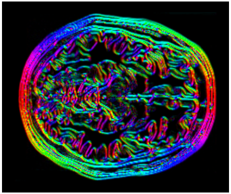
Definition: The **Directional Total Variation (dTV)** of u is

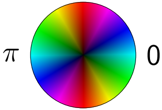
$$\mathrm{dTV}(u) := \sum_i \|[\mathbf{I} - \xi_i \xi_i^T] \nabla u_i\|, \quad 0 \le \|\xi_i\| \le 1$$

Ehrhardt and Betcke '16, related to Kaipio et al. '99, Bayram and Kamasak '12

- If $c > 0$, $\|\xi_i\|^2 \le 1 - c$, then $c\,\mathrm{TV} \le \mathrm{dTV} \le \mathrm{TV}$.
- If $\xi_i = 0$, then $\mathrm{dTV} = \mathrm{TV}$.
- $\xi_i = \frac{\nabla v_i}{\|\nabla v_i\|_\eta}, \quad \|\nabla v_i\|_\eta^2 = \|\nabla v_i\|^2 + \eta^2, \quad \eta > 0$







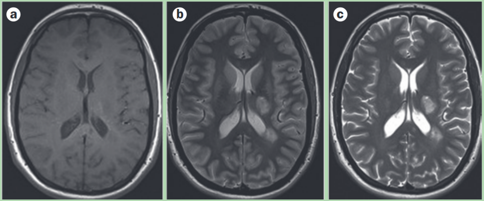
Application Examples

Multi-Modality Imaging Examples

PET-MR	Multi MRI	Spectral CT	Hyper + optical	X-ray + optical

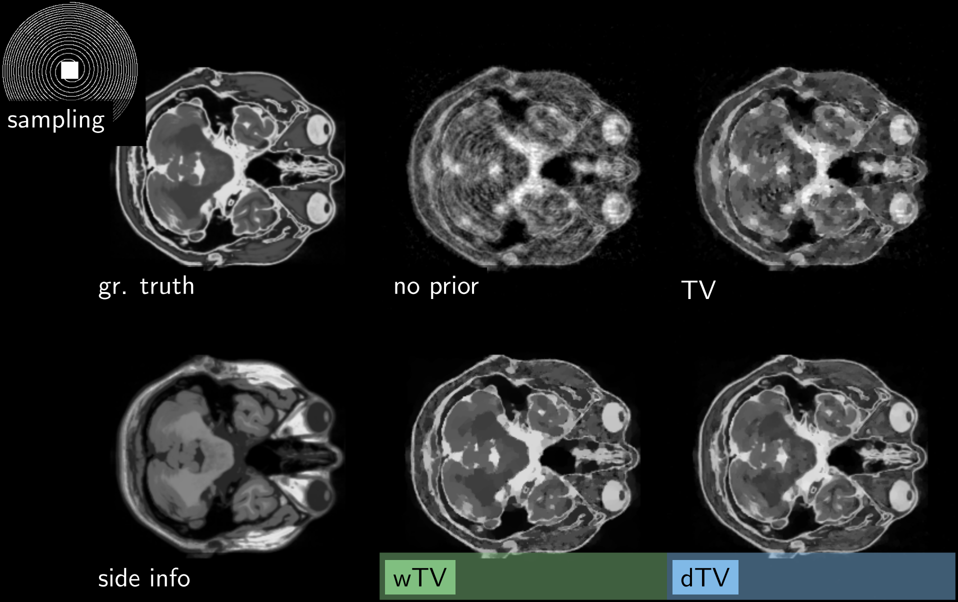
Multi-Sequence MRI

Ehrhardt and Betcke, SIAM J. Imaging Sci., vol. 9, no. 3, pp. 1084–1106, 2016.



Joint work with:
Computer Science: M. Betcke (UCL)

Multi-Sequence MRI Results



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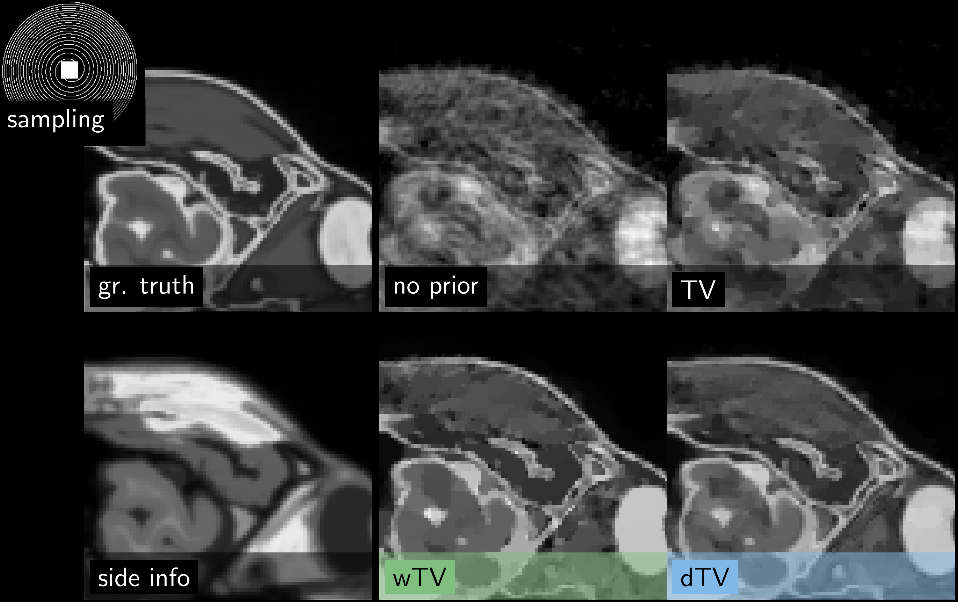

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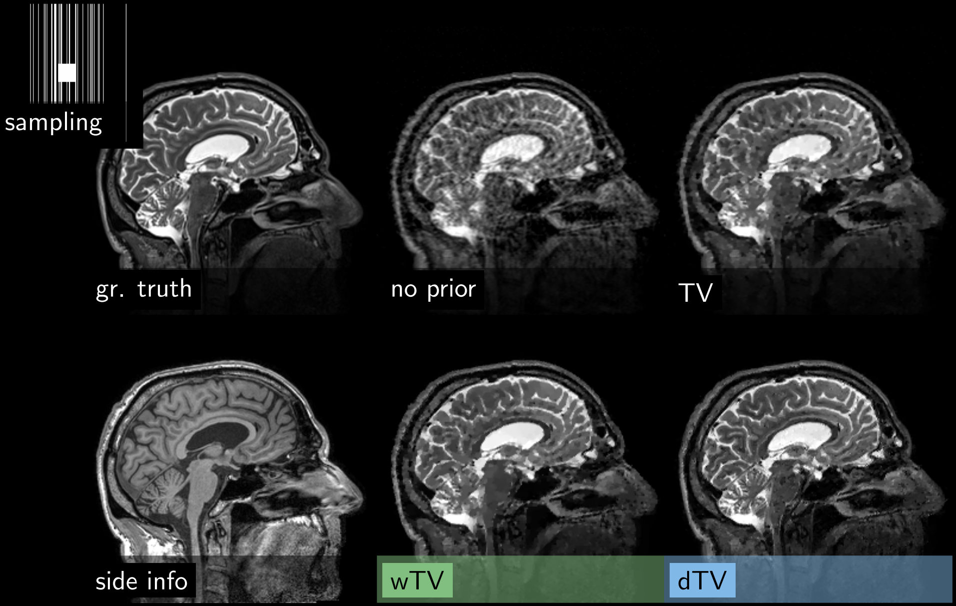

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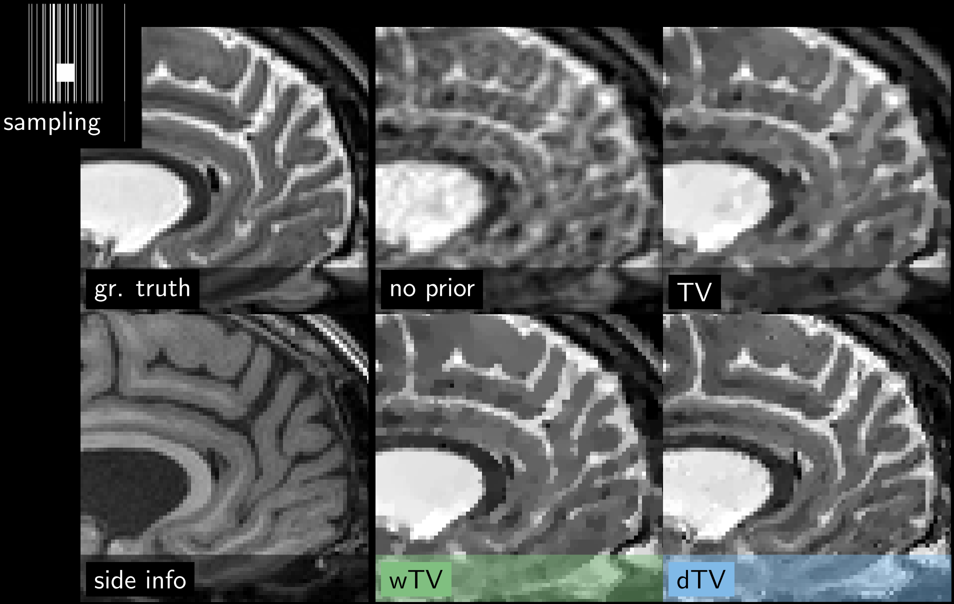
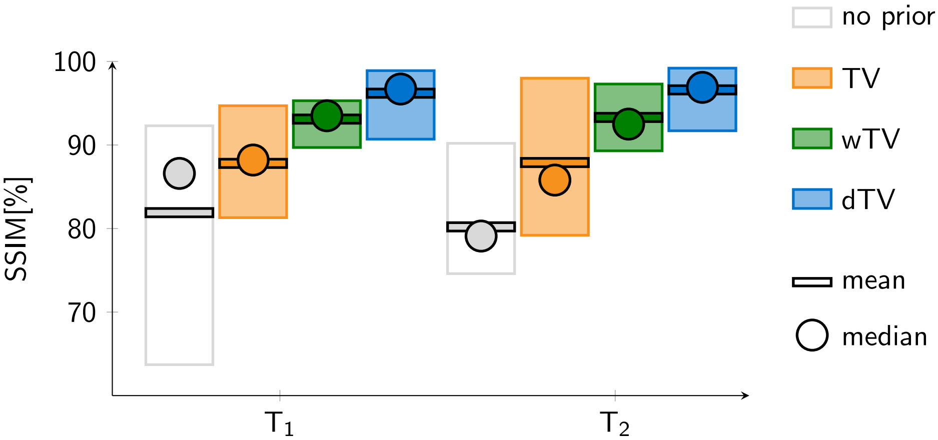


Quantitative Results



- Range (min, max), mean and median over 12 data sets

Multi-Modality Imaging Examples

PET-MR

PET-MR

Ehrhardt et al., Phys. Med. Biol. (in press), 2019
Ehrhardt et al., Proceedings of SPIE, vol. 10394, pp. 1–12, 2017

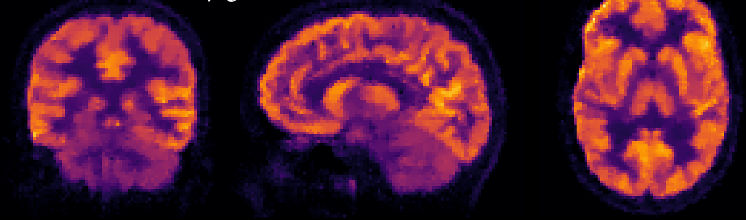
Joint work with:
Mathematics: A. Chambolle (École Polytechnique, France), P. Richtárik (KAUST, Saudi Arabia), C. Schönlieb (Cambridge)
Medical Physics: P. Markiewicz (UCL),
Neurology: J. Schott (UCL)

PET-MR Results

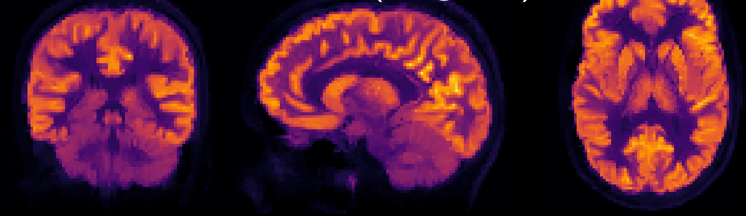
Reconstruction model:

$$\min_u \left\{ \mathrm{KL}(\mathbf{A}u + r; b) + \lambda \mathcal{J}(u) + \imath_{\geq 0}(u) \right\}$$

Total Variation, $\mathcal{J} = \mathrm{TV}$



Directional Total Variation (using MRI), $\mathcal{J} = \mathrm{dTV}$



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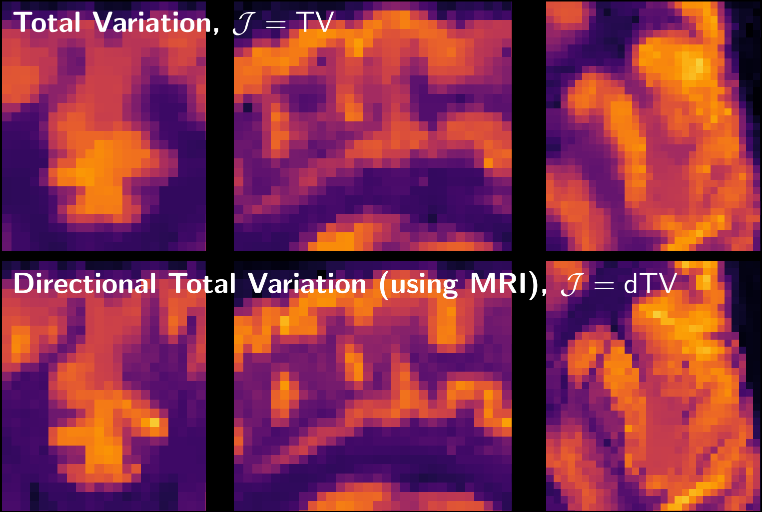
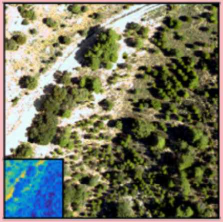
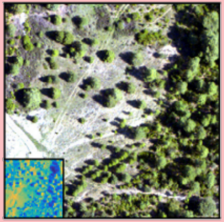
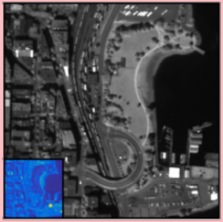


Multi-Modality Imaging Examples

PET-MR	Multi MRI	Spectral CT	Hyper + optical	X-ray + optical

Image fusion in remote sensing

Bungert et al., Inverse Probl., vol. 34, no. 4, p. 044003, 2018

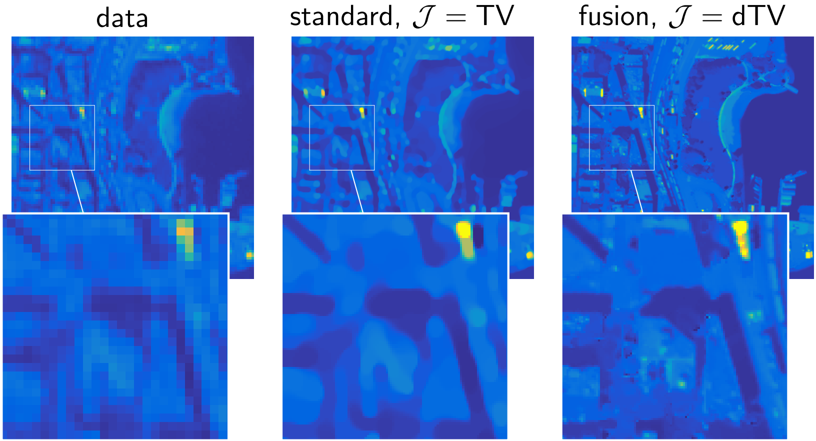




Joint work with:
Mathematics: L. Bungert (Erlangen, Germany), R. Reisenhofer (Vienna, Austria), J. Rasch (Berlin, Germany), C. Schönlieb (Cambridge),
Biology: D. Coomes (Cambridge)

Standard regularization versus image fusion

Reconstruction model:

$$\min_u \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \imath_{\geq 0}(u) \right\}$$



Blind versus non-blind image fusion

reconstruction model:

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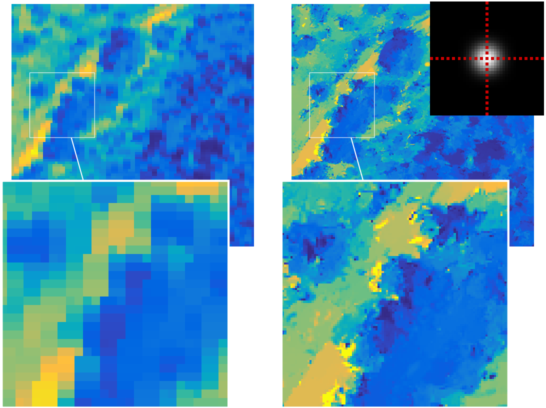
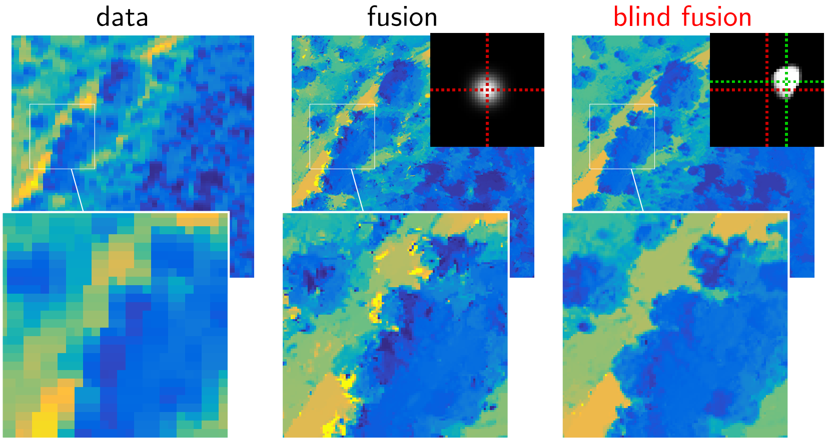
data fusion



Blind versus non-blind image fusion

Blind reconstruction model:

$$\min_{u,k} \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \imath_{\geq 0}(u) + \imath_S(k) \right\}$$



Conclusions and Outlook

Summary:

- **Multi-Modality Imaging** examples: PET-MR, multi-sequence MRI, spectral CT, Hyper + optical, X-ray + optical
- **Mathematical Models** to exploit synergies between modalities
- **Examples:** indeed often $1 + 1 > 2$!

Future:

- Which modalities **complement** each other best?
- Multi-modality imaging can help to **lower dose**, **increase resolution** ...
- **Expertise** in image / video processing, compressed sensing, machine learning ...