

$$1 + 1 > 2?$$

# Getting More Out of Multi-Modality Imaging

Matthias J. Ehrhardt

September 26, 2019

Institute for  
Mathematical Innovation



UNIVERSITY OF  
**BATH**

**EPSRC**

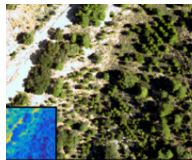
Engineering and Physical Sciences  
Research Council



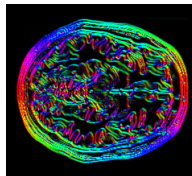
THE FARADAY  
INSTITUTION

# Outline

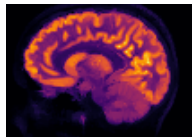
1) Motivation: Examples of Multi-Modality Imaging (**Why?**)



2) Mathematical Models for Multi-Modality Imaging (**How?**)



3) Application Examples: Remote Sensing and Medical Imaging ( $1 + 1 > 2?$ )

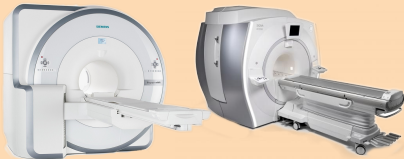


Motivation: Examples of Multi-Modality Imaging

# Multi-Modality Imaging Examples

## PET-MR

### PET-MR (and PET-CT, SPECT-MR, SPECT-CT)



Combine **anatomical (MRI)** and **functional (PET)** information

7 **clinical scanners** in UK

Currently images are just **overlayed**

**Challenge:** Reduce scanning time, increase image quality, lower dose

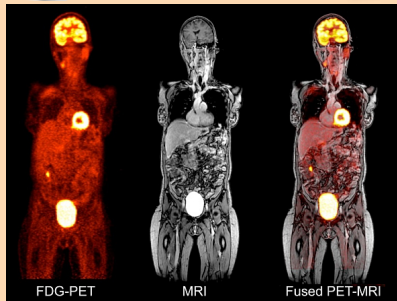


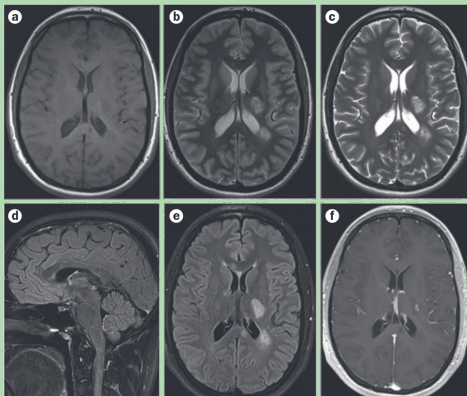
image: [Sheth and Gee, 2012](#)

# Multi-Modality Imaging Examples

PET-MR

Multi MRI

## Multi-Sequence MRI



**pre-contrast**

$T_1$ -weighted (a),  
dual-echo  $T_2$  (b, c)

**post-contrast**

2D  $T_2$  FLAIR (d, e),  
 $T_1$ -weighted (f)

**Standardized  
MRI protocol**  
for multiple sclerosis

**6 scans, total 30 min**

Rovira et al., Nature Reviews Neurology, 2015

**Challenge: Reduce scanning time**

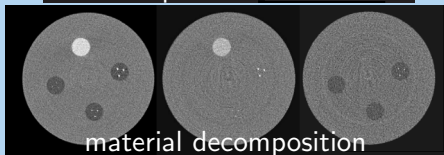
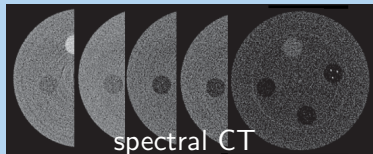
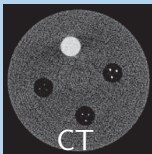
# Multi-Modality Imaging Examples

PET-MR

Multi MRI

Spectral CT

## Spectral CT



images:

Shikhaliyev and Fritz, 2011

**Acquisition:** energy resolved measurements

**Combination:** material information

**Challenge:** Low dose / high noise in some channels

# Multi-Modality Imaging Examples

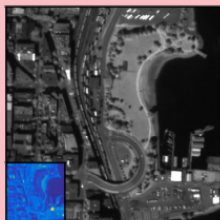
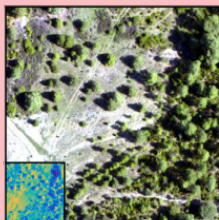
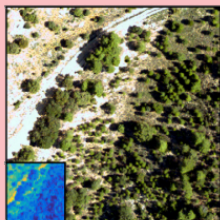
PET-MR

Multi MRI

Spectral CT

Hyper  
+ optical

## Image fusion in remote sensing



**Acquisition:** low resolution hyperspectral data (127 channels,  $1m \times 1m$ ) and high resolution photograph ( $0.25m \times 0.25m$ ) acquired **on plane or satellite**, e.g. by NERC Airborne Research & Survey Facility

**Challenge:** get best of both worlds—high spatial and spectral resolution

# Multi-Modality Imaging Examples

PET-MR

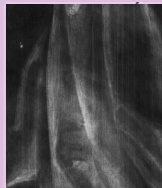
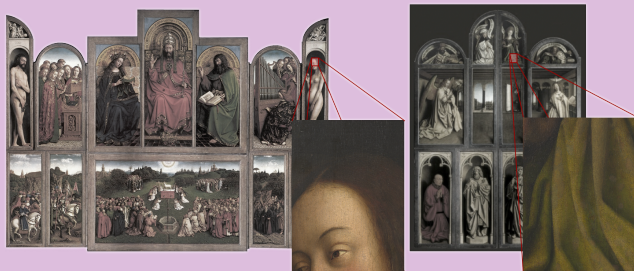
Multi MRI

Spectral CT

Hyper  
+ optical

X-ray  
+ optical

## X-ray separation for art restoration [Deligiannis et al. 2017](#)



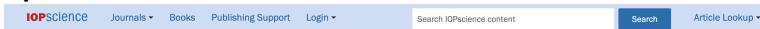
**Acquisition:** photographs and x-ray images

**Challenge:** separate the x-rays of the doors



# Fairly Large Field

- ▶ Regular **sessions at major conferences**: Applied Inverse Problems, SIAM Imaging
- ▶ **Symposium** in Manchester in 3-6 Nov 2019
- ▶ **Special Issue** in IOP Inverse Problems



## Inverse Problems

### Special issue on Joint Reconstruction and Multi-Modality/Multi-Spectral Imaging

*Inverse Problems* is pleased to announce the following upcoming special issue, which is now open for submissions via our submissions [page](#). We also kindly ask you to distribute this call among all colleagues who might be interested in submitting their work.

#### Guest editors

- [Simon Arridge](#) University College London, UK
- [Martin Burger](#) Universität Münster, Germany
- [Matthias Ehrhardt](#) University of Cambridge, UK

#### JOURNAL LINKS

[Submit an article](#)

[About the journal](#)

[Editorial Board](#)

[Author guidelines](#)

[Review for this journal](#)

[Publication charges](#)

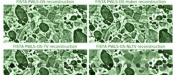
[News and editorial](#)

[Journal collections](#)

- ▶ **Collaborative Software Projects: CCPi (Phil Withers) and CCP PETMR**

CCPi  
Tomographic  
Imaging

[Home](#) [Projects](#) [Events](#) [Downloads](#) [Funding](#) [Gallery](#) [About Us](#)



Collaborative Computational Project in Tomographic Imaging aims to provide the UK tomography community with a toolbox of algorithms that increases the quality and level of information that can be extracted by computed tomography. Chaired by Prof Philip Withers (University of Manchester), co-edited and hosted by staff in the Scientific Computing Department (STC), led by a working group of experimental and theoretical academics with links to Diamond Light Source, DSI and Industry. [Read more here](#)

**NEWS FLASH:** CCPi ToSick Event at University of Southampton - 13-15 September [website](#)



Synergistic PET-MR Reconstruction

[Welcome](#) [Reports](#) [Software](#) [Events](#) [Funded Exchange](#) [Educational Resources](#) [Contacts](#)

[Home](#) [Synergistic Reconstruction Symposium](#)

### Synergistic Reconstruction Symposium

Date: Sunday, November 3, 2019 - 08:00 to Wednesday, November 6, 2019 - 18:00

[Update](#)

CCP PET-MR

Collaborative Computational Project in Positron Emission Tomography and Magnetic Resonance Imaging

#### Future Events

- [Bi-weekly software team](#)
- [FRI, 04/10/2019 - 10:30 to 12:00](#)
- [Bi-weekly software team](#)
- [FRI, 18/10/2019 - 10:30 to 12:00](#)
- [STH Users and Developer's Meeting at UCL 2019](#)

# Mathematical Models for Multi-Modality Imaging

# Image Reconstruction

## Variational Approach:

$$u^* \in \arg \min_u \left\{ \mathcal{D}(\mathbf{A}u, b) + \alpha \mathcal{J}(u) + \iota_{\mathcal{C}}(u) \right\}$$

**A forward operator** (often but not always linear),  
e.g. Radon transform

**D data fit**, e.g. least-squares  $\mathcal{D}(\mathbf{A}u, b) = \frac{1}{2} \|\mathbf{A}u - b\|^2$ ,  
Kullback–Leibler divergence

$$\mathcal{D}(\mathbf{A}u, b) = \int \mathbf{A}u - b + b \log(b/\mathbf{A}y)$$

**J regularizer**, e.g. total variation

$$\mathcal{J}(u) = \text{TV}(u) := \sum_i |\nabla u_i| \quad \text{Rudin et al., 1992}$$

$\iota_{\mathcal{C}}$  **constraints**, e.g. nonnegativity

# Image Reconstruction

## Variational Approach:

$$u^* \in \arg \min_u \left\{ \mathcal{D}(\mathbf{A}u, b) + \alpha \mathcal{J}(u) + \iota_{\mathcal{C}}(u) \right\}$$

**A forward operator** (often but not always linear),  
e.g. Radon transform

**D data fit**, e.g. least-squares  $\mathcal{D}(\mathbf{A}u, b) = \frac{1}{2} \|\mathbf{A}u - b\|^2$ ,  
Kullback–Leibler divergence

$$\mathcal{D}(\mathbf{A}u, b) = \int \mathbf{A}u - b + b \log(b/\mathbf{A}y)$$

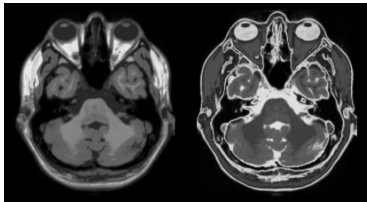
**J regularizer**, e.g. total variation

$$\mathcal{J}(u) = \text{TV}(u) := \sum_i |\nabla u_i| \quad \text{Rudin et al., 1992}$$

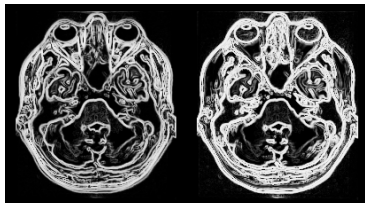
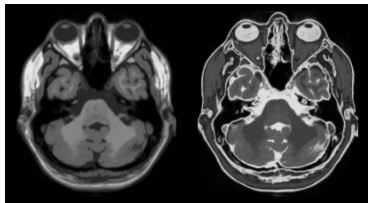
$\iota_{\mathcal{C}}$  **constraints**, e.g. nonnegativity

**How to include information from other modalities?**

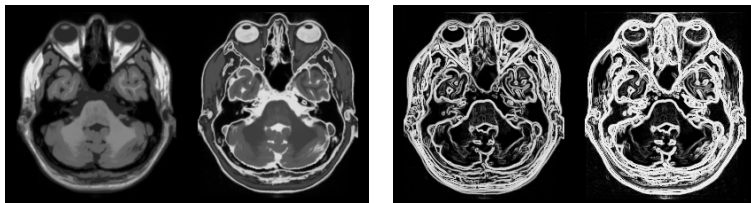
# Modelling Structural Similarity



# Modelling Structural Similarity



# Modelling Structural Similarity



**Definition:** The **Weighted Total Variation (wTV)** of  $u$  is

$$\text{dTV}(u) := \sum_i w_i \|\nabla u_i\|, \quad 0 \leq w_i \leq 1$$

See e.g. [Ehrhardt and Betcke '16](#)

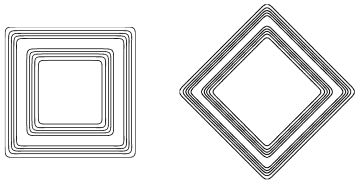
- ▶ If  $c > 0$ ,  $c < w_i$ , then  $c \text{TV} \leq \text{wTV} \leq \text{TV}$ .
- ▶ If  $w_i = 1$ , then  $\text{wTV} = \text{TV}$ .
- ▶  $w_i = \frac{\eta}{\|\nabla v_i\|_\eta}$ ,  $\|\nabla v_i\|_\eta^2 = \|\nabla v_i\|^2 + \eta^2$ ,  $\eta > 0$

# Modelling Structural Similarity

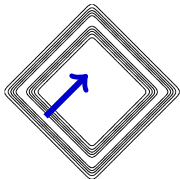
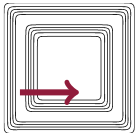




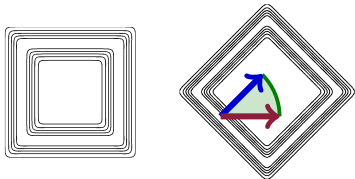
# Modelling Structural Similarity



# Modelling Structural Similarity

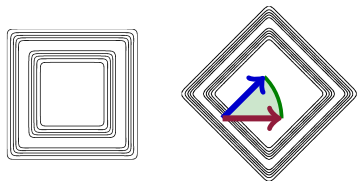


# Modelling Structural Similarity



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

# Modelling Structural Similarity

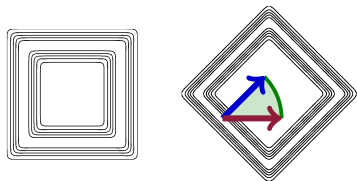


$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

**Definition:** Two images  $u$  and  $v$  are said to have **parallel level sets** or are **structurally similar** (denoted by  $u \sim v$ ) if  $\theta = 0$  or  $\theta = \pi$ , i.e.

$$\nabla u \parallel \nabla v \quad \text{i.e. } \exists \alpha \text{ such that } \nabla u = \alpha \nabla v.$$

# Modelling Structural Similarity



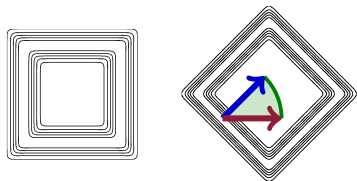
$$\langle \nabla u, \nabla v \rangle = \cos(\theta) \|\nabla u\| \|\nabla v\|$$

**Definition:** Two images  $u$  and  $v$  are said to have **parallel level sets** or are **structurally similar** (denoted by  $u \sim v$ ) if  $\theta = 0$  or  $\theta = \pi$ , i.e.

$$\nabla u \parallel \nabla v \quad \text{i.e. } \exists \alpha \text{ such that } \nabla u = \alpha \nabla v.$$

- ▶ Dominant idea in this field
  - ▶ Parallel Level Set Prior, e.g. [Ehrhardt and Arridge '14](#)
  - ▶ Directional Total Variation, e.g. [Ehrhardt and Betcke '16](#)
  - ▶ Total Nuclear Variation, e.g. [Knoll et al. '16](#)
  - ▶ Coupled Bregman iterations, e.g. [Rasch et al. '18](#)
- ▶ Others are: joint sparsity (e.g. wTV), joint entropy, ...

# Modelling Structural Similarity



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

**Definition:** Two images  $u$  and  $v$  are said to have **parallel level sets** or are **structurally similar** (denoted by  $u \sim v$ ) if  $\theta = 0$  or  $\theta = \pi$ , i.e.

$$\nabla u \parallel \nabla v \quad \text{i.e. } \exists \alpha \text{ such that } \nabla u = \alpha \nabla v.$$

- ▶ Dominant idea in this field
  - ▶ Parallel Level Set Prior, e.g. [Ehrhardt and Arridge '14](#)
  - ▶ **Directional Total Variation**, e.g. [Ehrhardt and Betcke '16](#)
  - ▶ Total Nuclear Variation, e.g. [Knoll et al. '16](#)
  - ▶ Coupled Bregman iterations, e.g. [Rasch et al. '18](#)
- ▶ Others are: joint sparsity (e.g. wTV), joint entropy, ...

# Directional Total Variation

- ▶ Note that if  $\|\nabla v\| = 1$ , then

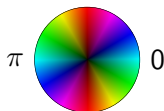
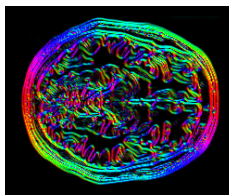
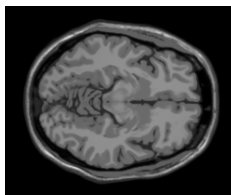
$$u \sim v \Leftrightarrow \nabla u - \langle \nabla u, \nabla v \rangle \nabla v = 0$$

**Definition:** The **Directional Total Variation (dTV)** of  $u$  is

$$\text{dTV}(u) := \sum_i \|\mathbf{I} - \xi_i \xi_i^T\| \|\nabla u_i\|, \quad 0 \leq \|\xi_i\| \leq 1$$

Ehrhardt and Betcke '16, related to Kaipio et al. '99, Bayram and Kamasak '12

- ▶ If  $c > 0$ ,  $\|\xi_i\|^2 \leq 1 - c$ , then  $c \text{TV} \leq \text{dTV} \leq \text{TV}$ .
- ▶ If  $\xi_i = 0$ , then  $\text{dTV} = \text{TV}$ .
- ▶  $\xi_i = \frac{\nabla v_i}{\|\nabla v_i\|_\eta}$ ,  $\|\nabla v_i\|_\eta^2 = \|\nabla v_i\|^2 + \eta^2$ ,  $\eta > 0$



## Application Examples



# Multi-Modality Imaging Examples

PET-MR

Multi MRI

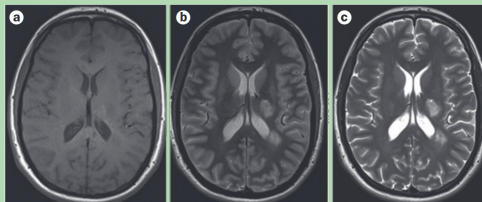
Spectral CT

Hyper  
+ optical

X-ray  
+ optical

## Multi-Sequence MRI

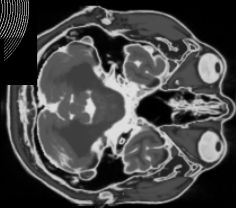
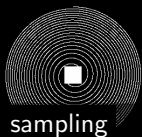
Ehrhardt and Betcke, *SIAM J. Imaging Sci.*, vol. 9, no. 3, pp. 1084–1106, 2016.



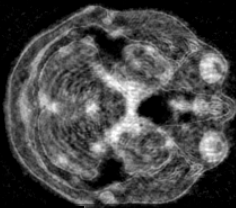
Joint work with:

**Computer Science:** M. Betcke (UCL)

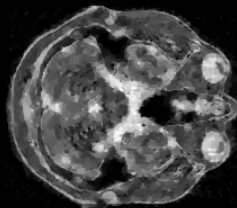
# Multi-Sequence MRI Results



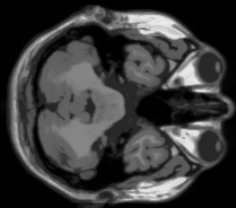
gr. truth



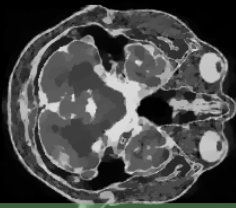
no prior



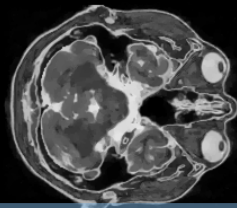
TV



side info

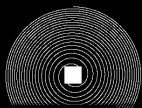


wTV

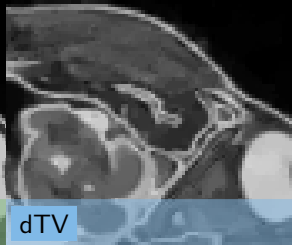
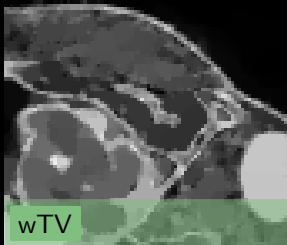
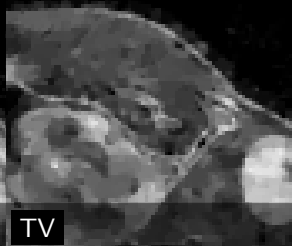
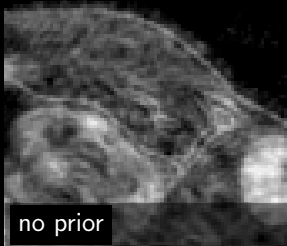


dTV

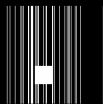
# Multi-Sequence MRI Results



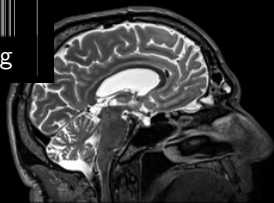
sampling



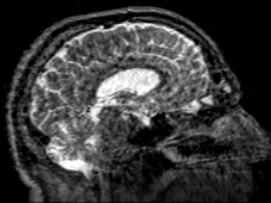
# Multi-Sequence MRI Results



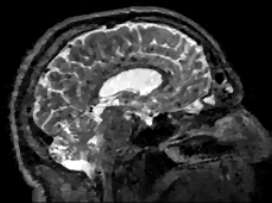
sampling



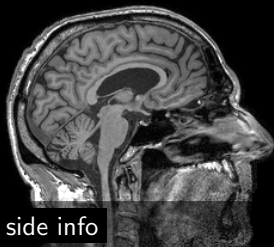
gr. truth



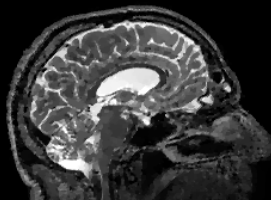
no prior



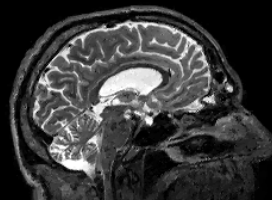
TV



side info



wTV



dTV

# Multi-Sequence MRI Results



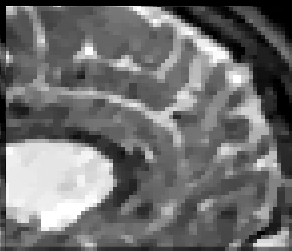
sampling



gr. truth



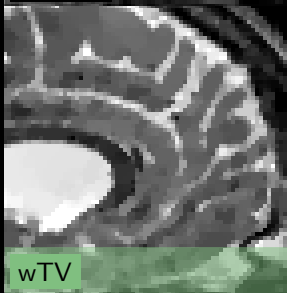
no prior



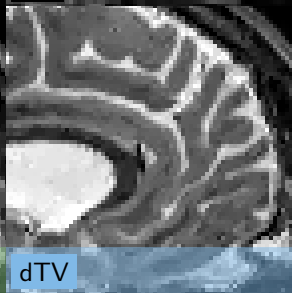
TV



side info

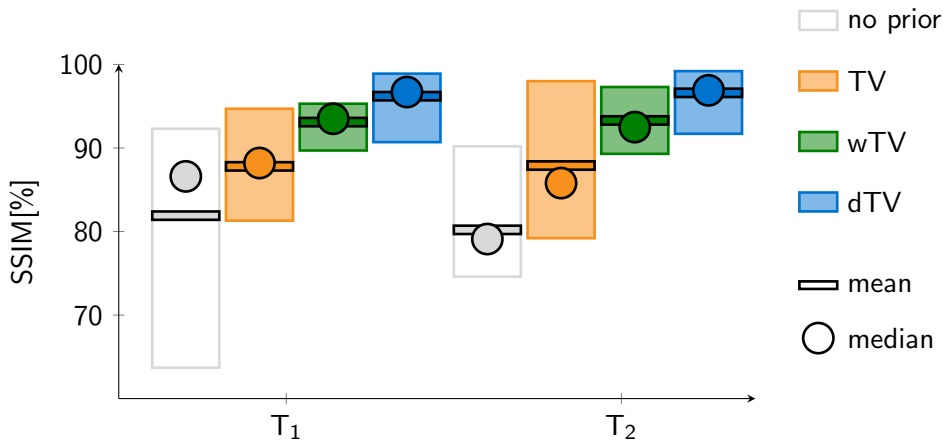


wTV



dTV

# Quantitative Results



► Range (min, max), mean and median over 12 data sets

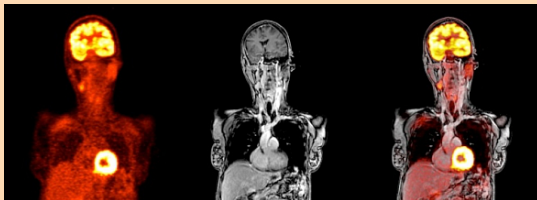
# Multi-Modality Imaging Examples

PET-MR

## PET-MR

Ehrhardt et al., Phys. Med. Biol. (in press), 2019

Ehrhardt et al., Proceedings of SPIE, vol. 10394, pp. 1–12, 2017



Joint work with:

**Mathematics:** A. Chambolle (École Polytechnique, France), P. Richtárik (KAUST, Saudi Arabia), C. Schönlieb (Cambridge)

**Medical Physics:** P. Markiewicz (UCL),

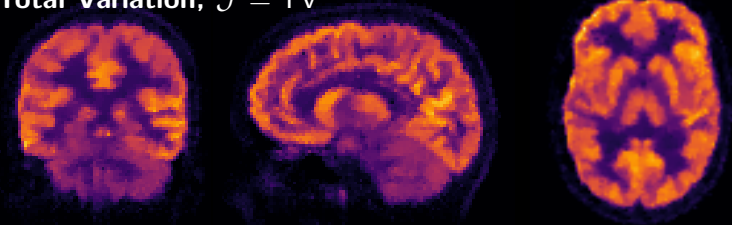
**Neurology:** J. Schott (UCL)

# PET-MR Results

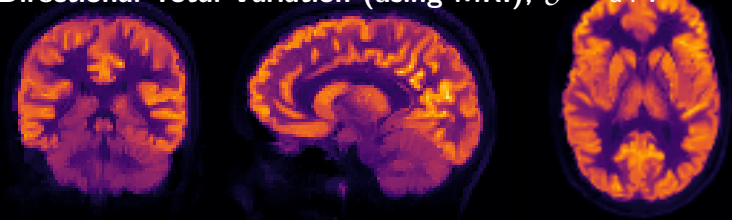
Reconstruction model:

$$\min_u \left\{ \text{KL}(\mathbf{A}u + r; b) + \lambda \mathcal{J}(u) + \nu_{\geq 0}(u) \right\}$$

Total Variation,  $\mathcal{J} = \text{TV}$



Directional Total Variation (using MRI),  $\mathcal{J} = \text{dTV}$



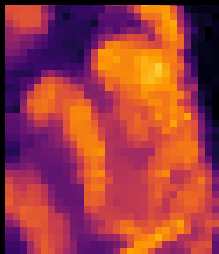
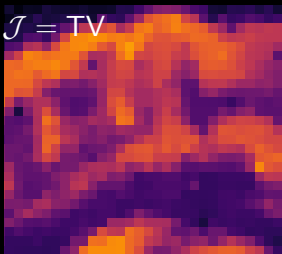
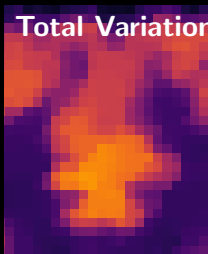


# PET-MR Results

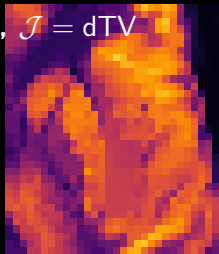
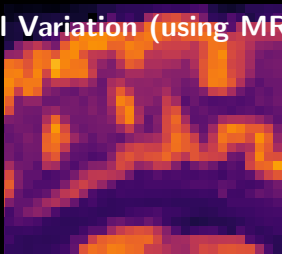
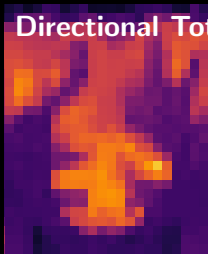
Reconstruction model:

$$\min_u \left\{ \text{KL}(\mathbf{A}u + r; b) + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$

Total Variation,  $\mathcal{J} = \text{TV}$



Directional Total Variation (using MRI),  $\mathcal{J} = \text{dTV}$



# Multi-Modality Imaging Examples

PET-MR

Multi MRI

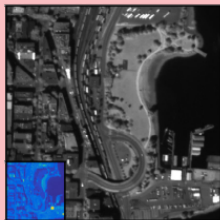
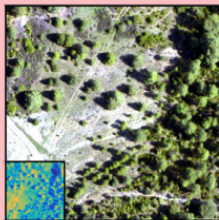
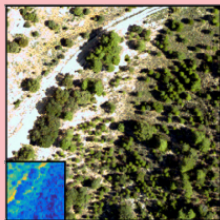
Spectral CT

Hyper  
+ optical

X-ray  
+ optical

## Image fusion in remote sensing

Bungert et al., *Inverse Probl.*, vol. 34, no. 4, p. 044003, 2018



Joint work with:

**Mathematics:** L. Bungert (Erlangen, Germany), R. Reisenhofer (Vienna, Austria), J. Rasch (Berlin, Germany), C. Schönlieb (Cambridge),

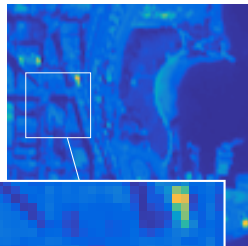
**Biology:** D. Coomes (Cambridge)

# Standard regularization versus image fusion

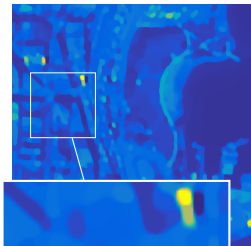
**Reconstruction model:**

$$\min_u \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$

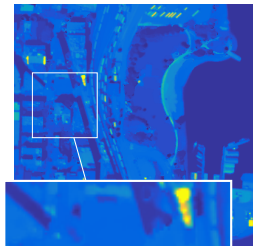
data



standard,  $\mathcal{J} = \text{TV}$



fusion,  $\mathcal{J} = \text{dTV}$

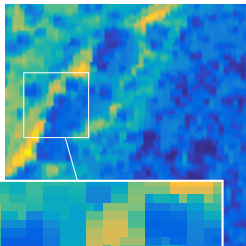


# Blind versus non-blind image fusion

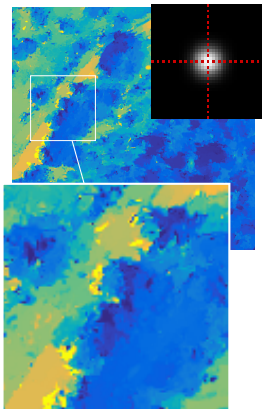
**reconstruction model:**

$$\min_u \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) \right\}$$

data



fusion

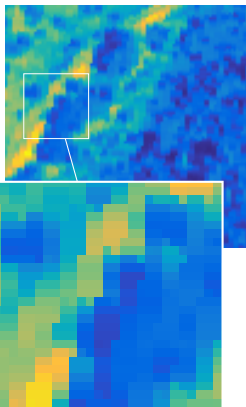


# Blind versus non-blind image fusion

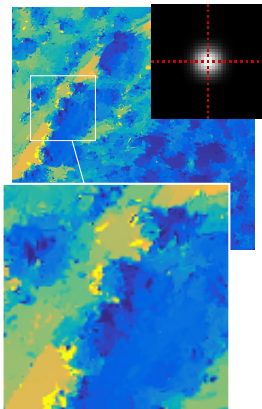
**Blind** reconstruction model:

$$\min_{u,k} \left\{ \frac{1}{2} \|\mathbf{S}(u * k) - v\|^2 + \lambda \mathcal{J}(u) + \iota_{\geq 0}(u) + \iota_S(k) \right\}$$

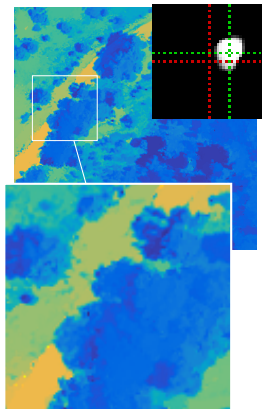
data



fusion



blind fusion



# Conclusions and Outlook

## Summary:

- ▶ **Multi-Modality Imaging** examples: PET-MR, multi-sequence MRI, spectral CT, Hyper + optical, X-ray + optical
- ▶ **Mathematical Models** to exploit synergies between modalities
- ▶ **Examples:** indeed often  $1 + 1 > 2!$

## Future:

- ▶ Which modalities **complement** each other best?
- ▶ Multi-modality imaging can help to **lower dose, increase resolution** ...
- ▶ **Expertise** in image / video processing, compressed sensing, machine learning ...

