Robust Image Reconstruction with Misaligned Structural Information

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24 March, 2022

Joint work with: Leon Bungert (Bonn, Germany)

The Leverhulme Trust

Engineering and Physical Sciences Research Council

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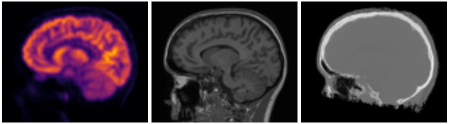
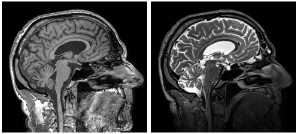
Multi-Modality Imaging

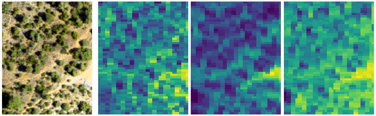


PET, MRI, CT
Ehrhardt et al. '15, Knoll et al. '16,
Schramm et al. '17, Mehranian et al. '18

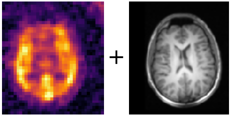


Multi-Contrast MRI
Bilgic et al. '11,
Ehrhardt and Betcke '16

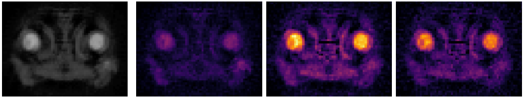
Color Photography
Möller et al. '14, Holt '14



Hyperspectral Remote Sensing
Möller et al. '12, Bungert et al. '18



hyperpolarized +
proton MR
Ehrhardt et al. '21



Spectral CT Kazantsev et al. '18

Multi-Modality Inverse Problems

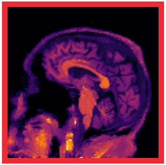
Single-Modality

Classic

$$Au = f$$

- Given f
- Recover u

Scherzer et al. '09

 $\longrightarrow$ 

Multi-Modality Inverse Problems

Single-Modality

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$$Au = f$$

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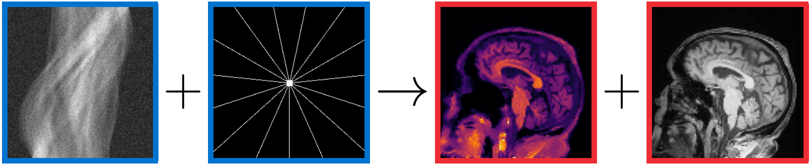
Multi-Modality

Joint Recon

$$Au = f \quad Bv = g$$

- Given f, g
- Recover u, v

Arridge, Ehrhardt, Thielemans '21



Multi-Modality Inverse Problems

Single-Modality | **Multi-Modality**

Classic

$$Au = f$$

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Scherzer et al. '09

Guided Recon

$$Au = f$$

- Given f, v
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Ehrhardt '21

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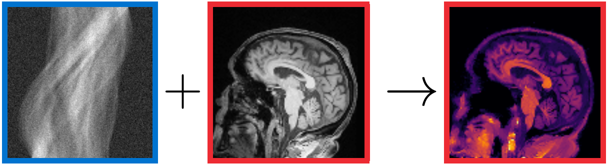
Arridge, Ehrhardt, Thielemans '21



Multi-Modality Inverse Problems

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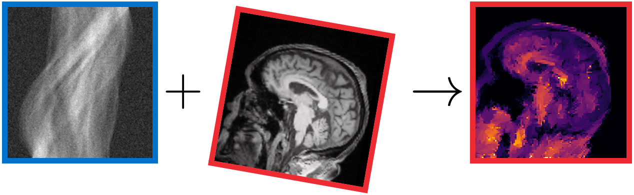


Our Goal: Make guided recon robust! Bungert and Ehrhardt '20

Variational Regularization

Approximate solution of $Au = f$ via

$$\hat{u} = \arg\min_u \left\{ D(Au, f) + \lambda \mathcal{R}(u) \right\}$$

- D: data fidelity, related to noise statistics, e.g.

$$\|Au - f\|_2^2$$

- $\mathcal{R}$: **regularizer**: penalizes unwanted features, e.g. total variation Rudin, Osher, Fatemi '92

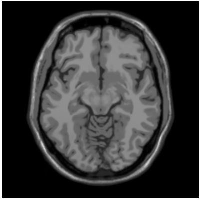
$$\mathrm{TV}(u) := \sum_i \|\nabla u_i\|$$

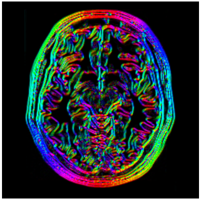
How to include "guide" into reconstruction?

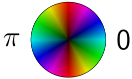
Directional Total Variation Ehrhardt and Betcke '16

$$\mathrm{dTV}(u) := \sum_i \|D_i \nabla u_i\|, \quad D_i = I - \xi_i \xi_i^T$$

- $\xi_i = \nabla v_i / \sqrt{\|\nabla v_i\|^2 + \eta^2}, \quad \eta > 0$







- If $0 < c, \|\xi_i\| \leq \sqrt{1-c}$, then $c\,\mathrm{TV} \leq \mathrm{dTV} \leq \mathrm{TV}$.
- If $\xi_i = 0$, then $\mathrm{dTV} = \mathrm{TV}$.

Directional Total Variation Ehrhardt and Betcke '16

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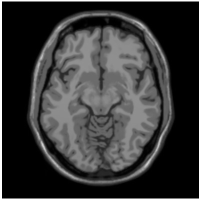
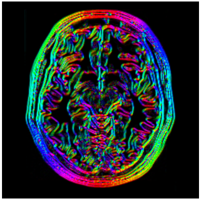
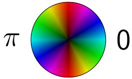






- If $0 < c, \|\xi_i\| \leq \sqrt{1-c}$, then $c\,\mathrm{TV} \leq \mathrm{dTV} \leq \mathrm{TV}$.
- If $\xi_i = 0$, then $\mathrm{dTV} = \mathrm{TV}$.
- Squared H^1 Kaipio et al. '99; constant ξ_i Bayram and Kamasak '12; Kongskov et al. '17; smoothed Ehrhardt et al. '16; Lenzen and Berger '15
- Concept generalizable, e.g. TGV Bredies et al. '10; Ehrhardt '21
- Other regularizers with guide, e.g. Bowsher et al. '04; Nuyts '07; Rasch et al. '18

Three-Step Method

1. Reconstruction:

$$\tilde{u} \in \arg\min_u D(Au; f) + \alpha \, \mathrm{TV}(u)$$

2. Registration:

$$\varphi^* \in \arg\max_\varphi MI(v, T_\varphi \tilde{u})$$

3. Guided Reconstruction

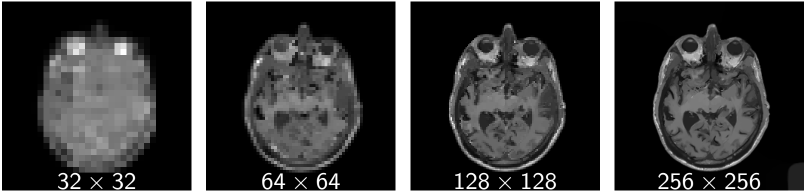
$$u^* \in \arg\min_u D(AT_{\varphi^*} u; f) + \alpha \, \mathrm{dTV}(u; v)$$

- Registration via maximizing mutual information Wells et al. '96
- $T_\varphi \tilde{u}$ is deformation of $\tilde{u}$ by φ: here affine transformation
- One could loop over the registration and guided recon steps

Joint Reconstruction-Registration

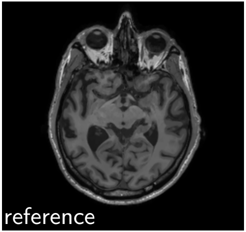
$$u^*, \varphi^* \in \arg\min_{u,\varphi} D(AT_\varphi u; f) + \alpha \, \mathrm{dTV}(u; v)$$

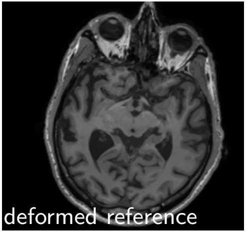
- Minimization via PALM with backtracking Bolte et al. '14
- Multi-resolution strategy necessary to avoid unwanted stationary points Modersitzki '09

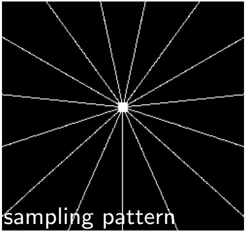


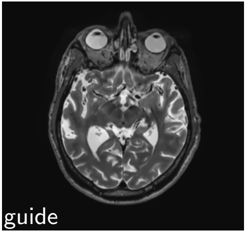
Numerical Results

Multi-Contrast MRI: Data

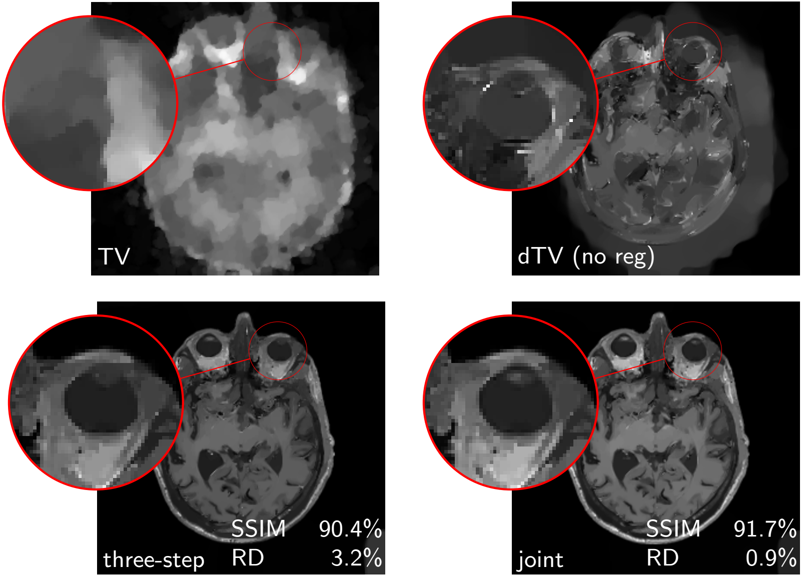




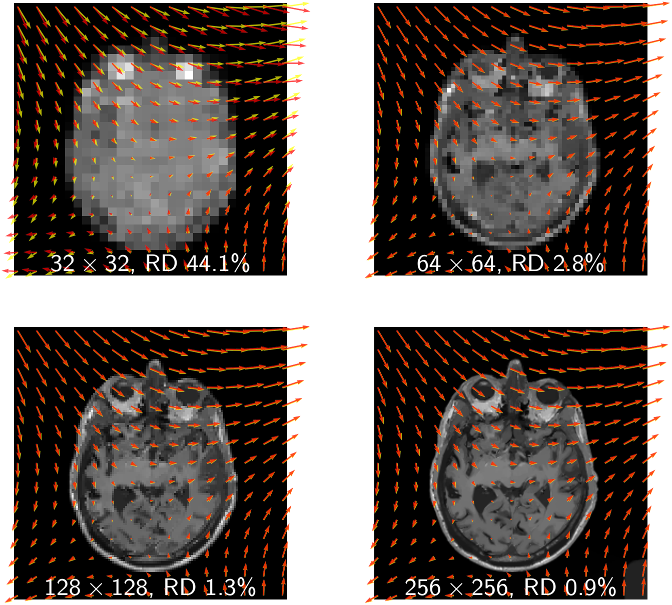




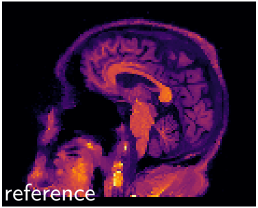
Multi-Contrast MRI: Results

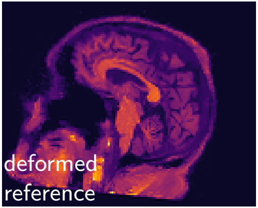


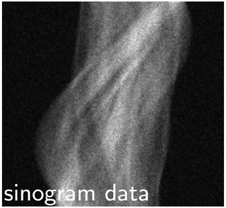
MRI: Vectorfields



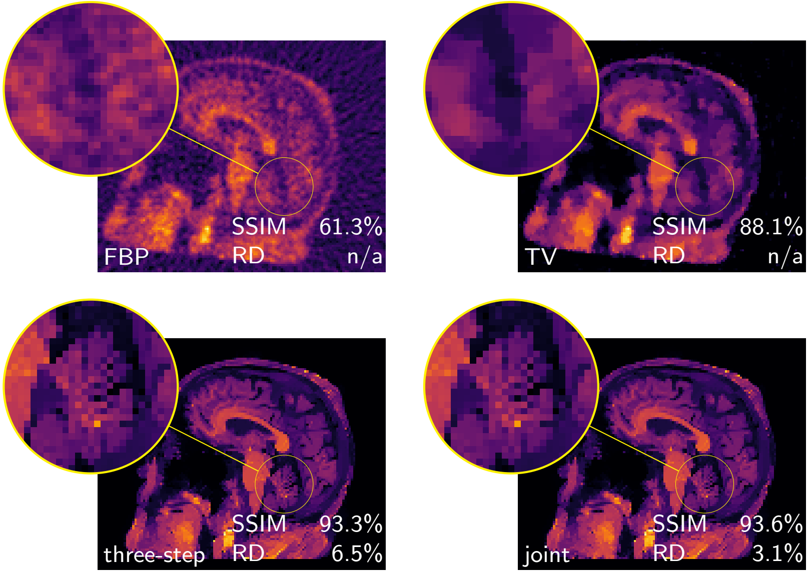
PET Data





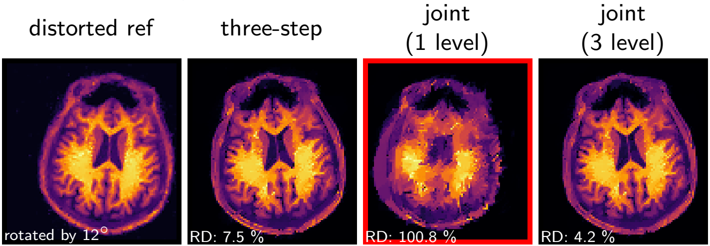


PET Results



SSIM 61.3%
FBP RD n/a
SSIM 88.1%
TV RD n/a
SSIM 93.3%
three-step RD 6.5%
SSIM 93.6%
joint RD 3.1%

Robustness to Large Rotations



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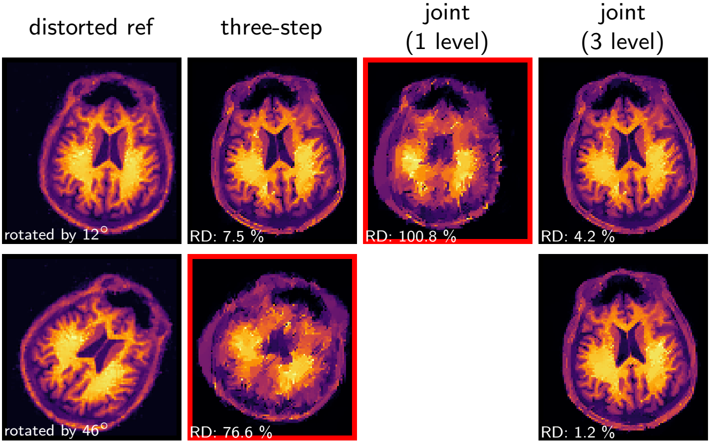

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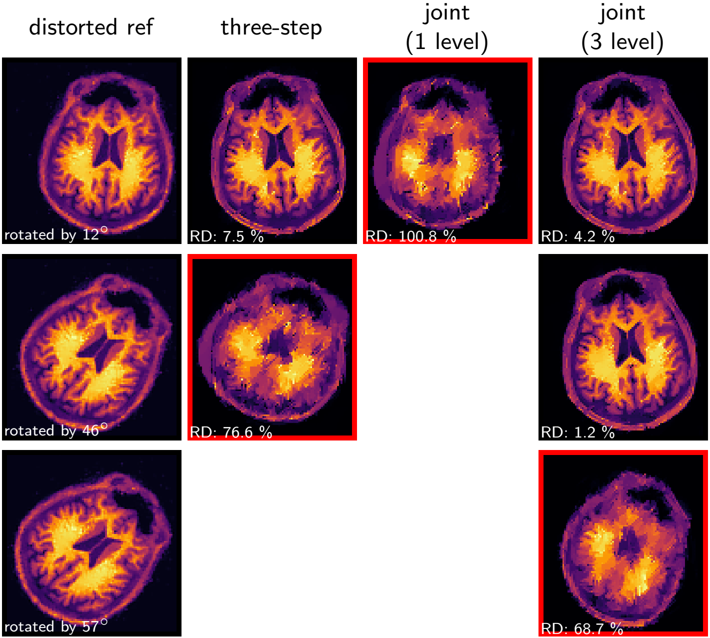

Conclusions

Multi-modality imaging

- joint or guided reconstruction
- variational models for joint structure exist, e.g. dTV
- sensitive to misregistration

Make guided reconstruction robust

- three-step approach (simpler)
- joint reconstruction-registration (better)